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Flux-induced SUSY-breaking soft terms on D7-D3 brane systems

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Abstract

We study the effect of RR and NSNS 3-form fluxes on the effective action of the worldvolume fields of Type IIB D7/D3-brane configurations. The D7-branes wrap 4-cycles Σ_4 on a local Calabi-Yau geometry. This is an extension of previous work on hep-th/0311241, where a similar analysis was applied to the case of D3-branes. Our present analysis is based on the D7- and D3-brane Dirac-Born-Infeld and Chern-Simons actions, and makes full use of the R-symmetries of the system, which allow us to compute explicitly results for the fields lying at the D3-D7 intersections. A number of interesting new properties appear as compared to the simpler case of configurations with only D3-branes. As a general result one finds that fluxes stabilize some or all of the D7-brane moduli. We argue that this is important for the problem of stabilizing Kähler moduli through non-perturbative effects in KKLT-like vacua. We also show that (0,3) imaginary self-dual fluxes, which lead to compactifications with zero vacuum energy, give rise to SUSY-breaking soft terms including gaugino and scalar masses, and trilinear terms. Particular examples of chiral MSSM-like models of this class of vacua, based on D3-D7 brane systems at orbifold singularities are presented.

1 Introduction

In the search for realistic string compactifications two important problems appear. On one hand, in e.g. familiar Calabi-Yau compactifications, one typically has a large number of perturbatively massless fields, namely the complex dilaton, Kähler and complex structure moduli, and possibly others (e.g. gauge bundle or brane moduli). Thus the string coupling as well as geometric data of the compactification remain undetermined at the perturbative level. A second problem, possibly related to the first one, is how to break supersymmetry in a controled manner if we start with e.g. a $N = 1$ supersymmetric compactification.

In the last few years it has been realized that a general compactification allows for an additional ingredient, not considered previously, namely field strength fluxes for the internal components of p-form supergravity fields [1, 2, 3, 4, 5, 6, 7, 8, 9]. Interestingly, this new ingredient may have an important bearing on those two problems. The case of NSNS/RR 3-form field strength fluxes in Type IIB CY orientifold compactifications (or more generally, F/M-theory on CY fourfolds with 4-form flux) has been studied in particular detail (see e.g. [2, 4, 6]). The presence of the fluxes induces a non-trivial warp factor in the geometry, as well as a non-trivial source for the RR 4-form potential. However it has been shown that, due to the presence of O3-planes in the IIB construction (or more generally, the F/M-theory tadpole from the Euler characteristic of the fourfold [10]), the Type IIB 10-dimensional equations of motion can be solved consistently with 4d Minkowski space, if the internal 3-form flux is imaginary self-dual, ISD (analogously, if the F/M-theory 4-form flux is self-dual).

The fact that the (quantised) fluxes should be ISD to solve the equations of motion generically determines dynamically the value of the complex dilaton τ as well as all complex structure moduli, while Kähler moduli are not stabilized. For adequate choices of fluxes, the complex dilaton may be fixed at a perturbative value, so that one expects the description of the compactifications in terms of classical supergravity to be a reliable approximation (in the large volume limit in Kähler moduli space). It has further been argued that non-perturbative effects depending on the Kähler moduli, which are generically present in this kind of compactification, like non-perturbative superpotentials from strong infrared dynamics of gauge sectors on the D7-branes or from euclidean D3-brane instantons, may determine also all the Kähler moduli [11] (possibly at large volume). This has been further explored in explicit models in [12, 13], with positive results in a sizeable number of examples [12]. This setting thus provides the first examples of tractable string compactifications with all dilaton, Kähler and

complex structure moduli determined dynamically.

Field strength fluxes may also be important for the second problem mentioned above, the breaking of supersymmetry, since the generic choice of fluxes in a compactification is in fact non-supersymmetric (although many examples with supersymmetry preserving fluxes have been constructed). In particular the field theories inside Dp-branes located in the CY compactification, which are supersymmetric in the absence of fluxes, may get SUSY-breaking soft terms in the presence of fluxes. This is particularly interesting since it is possible to embed chiral gauge sectors relatively close to the (MS)SM in D-brane configurations in the presence of 3-form fluxes [14, 15, 16]. In the case of gauge sectors localized on D3- (or anti-D3-) branes, such terms have been recently computed in [17, 18, 19]. In particular it was found in [18] (see also [19]) that only imaginary anti-selfdual (IASD) fluxes gives rise to SUSY-breaking soft terms on the worldvolume of D3-branes [1]. This is unfortunate because, as we mentioned, only ISD fluxes solve the equations of motion. Thus, in order to have non-vanishing soft-terms, we either add IASD fluxes, and hence do not solve the equations of motion at this level (hoping that other effects would perhaps stabilize the compactification), or else we include anti-D3-branes in the compactification, thus breaking supersymmetry in a less controlled fashion. The latter is certainly an interesting possibility, as advocated in [18], since anti-D3-branes turn out to be an important ingredient in the proposal in [11] to obtain deSitter vacua in string theory.

In any event, it would be interesting to have Type IIB CY orientifold compactifications (or more generally F-theory compactifications on CY 4-folds) which have $N = 1$ supersymmetry in the absence of fluxes, and yield SUSY-breaking soft terms upon turning them on, and still obeying the Type IIB equations of motion. One of the motivations of the present paper is to provide examples of this class. Specifically we analyze the effect of 3-form field strength fluxes on the world-volume fields of D7/D3-brane configurations, extending the results for D3-branes presented in [18, 19]. We show that, in contrast with the D3-brane case, ISD fluxes do give rise to non-vanishing soft terms for the fields on the worldvolume of D7-branes.

This result is interesting for a number of reasons. In particular this shows that Type IIB CY orientifolds in the presence of ISD fluxes provide us with (to our knowledge) the first known class of string compactifications to 4d Minkowski space which solve the classical equations of motion and lead to non-trivial SUSY-breaking soft terms, in a

[1] On the other hand ISD do give rise to soft terms on the worldvolume of anti-D3-branes. The presence of these however break supersymmetry in a less controlled fashion.

controled manner. Apart from its theoretical interest, the structure of soft terms, if applied to realistic models with the SM living on D7-branes, may be of phenomenological relevance (see [20]).

Another application of our results concerns the proposal in [11], attempting to fix dynamically the Kähler moduli in type IIB orientifolds of this class, mentioned above. A potential problem in generating non-perturbative superpotentials from e.g. strong infrared dynamics on the gauge theory on D7-branes, is the possible presence of too much massless charged matter in the latter. Our results for soft terms involving D7-brane matter fields show that the presence of ISD fluxes generally give masses to all the D7-brane geometric moduli. This indicates that generically D7-brane vector-like matter is massive, thus allowing such non-perturbative superpotentials to appear [2]. This is also in agreement with recent results derived from the F-theory perspective in [21], and from generalized calibrations [22].

Our results are based on an expansion of the relevant D7- and D3-brane Dirac-Born-Infeld plus Chern-Simons (DBI+CS) actions in the presence of fairly general type IIB closed string backgrounds. Since D-branes are mainly sensitive to the local geometry around them, our results are derived in an expansion around the location of D7- and D3-branes. Since D7-branes wrap a 4-cycle Σ_4 in the internal space, we center on the simple situation of D7-branes wrapping T^4 or $K3$ in a local Calabi-Yau $\Sigma_4 \times C$. The effect of fluxes on the fields living at D3-D7 (or $\overline{D3}-D7$) intersections is more involved, but we explicitly derive it in full generality using (super)symmetry arguments. Finally, all results are compared, showing full agreement, with an analysis based on the use of the 4d effective action [23].

The paper is organized as follows. After reviewing the results of [18] in section 2, we compute the soft terms for the fields on the worldvolume of D7's in section 3, checking also that our ansatz for the closed string backgrounds solves the equations of motion. In sections 4 (resp. 6) we make use of the local geometric symmetries of $D3$-$D7$ (resp. $\overline{D3}-D7$) configurations to carry out the computation of the soft terms for fields living at the $D3$-$D7$($\overline{D3}$-$D7$) intersections. Examples of soft terms obtained for different classes of ISD and IASD fluxes are described in section 5.

Some of the results obtained both for $D7-D3$-brane systems may be understood from the effective low energy $N=1$ supergravity action [23, 24, 25]. In particular, we show in section 7 that fluxes correspond to non-vanishing expectation values for

[2]An important exception is D7-branes containing charged chiral fermions. This suggests that the D7-branes responsible for Kähler moduli stabilization should not correspond to the D7-branes on which we plan to embed the SM, but to some non-chiral D7-brane sector.

the auxiliary fields of the dilaton and Kähler moduli. We show that the soft terms obtained in section 3 for ISD $(0,3)$ fluxes may be understood as arising from a non-vanishing auxiliary field $F_T \neq 0$ for the overall Kähler modulus T. This is in agreement with previous results in [18, 19]. A noteworthy property is that the bosonic soft terms induced by ISD fluxes naturally combine with the SUSY scalar potential into a positive definite scalar potential. This fact, which appears already in section 3 from the DBI+CS action in the presence of fluxes, has a simple interpretation also from the effective $N = 1$ supergravity low-energy action, as explained in section 7. We also show in that section that analogous soft terms are expected for the fields lying at the intersections of $D7$-branes wrapping different 4-cycles.

In section 8 we argue that many of the results obtained for D7-branes wrapping 4-tori are still valid when D7-branes wrap curved 4-cycles, in particular $K3$, and describe the explicit soft terms in this case. We also argue that similar patterns of soft terms are also obtained in more involved situations, where the normal bundle to the 4-cycle is non-trivial. For instance, in cases with multiple D7-brane geometric moduli, all of the latter are fixed in the presence of ISD backgrounds, in agreement with results in section 7.

Some particular examples and applications are described in section 9. In particular we present several examples of local D7/D3-brane configurations, with a semirealistic chiral gauge sector arising in the worldvolume of the D7-branes. The presence of ISD fluxes gives rise to phenomenologically interesting soft terms. Embedding this class of local configurations into a full F-theory compactification would give rise to semirealistic models in which calculable SUSY-breaking soft terms are induced. We also briefly discuss in section 9 the relevance of our results for the program in [11], as discussed above. We end up with some comments in section 10. Details of the computations are provided in an appendix.

2 Three-form fluxes and D3-branes

Let us first briefly review some of the main results in ref.[18] (see [17, 19] for related discussions). We consider D3-branes embedded in general Type IIB closed string backgrounds of the general form

$$\begin{aligned} ds^2 &= Z_1(x^m)^{-1/2}\eta_{\mu\nu}\, dx^\mu dx^\nu + Z_2(x^m)^{1/2}\, ds^2{}_{CY} \\ \tau &= \tau(x^m) \end{aligned}$$

$$
\begin{aligned}
G_3 &= \frac{1}{3!} G_{lmn}(x^m)\, dx^l dx^m dx^n \\
\chi_4 &= \chi(x^m)\, dx^0 dx^1 dx^2 dx^3 \\
F_5 &= d\chi_4 + *_{10}\, d\chi_4
\end{aligned} \tag{2.1}
$$

where $G_3 = F_3 - \tau H_3$ (with $F_3(H_3)$ being the RR(NSNS) flux), and $ds^2{}_{CY}$ denotes the metric in transverse space, in the absence of flux backreaction, i.e. the Ricci-flat metric of the underlying Calabi-Yau. Eventually we will request these backgrounds to solve the Type IIB supergravity equations of motion. Since D3-branes are located at a point in the six dimensions parametrized by x^m, it is natural to expand these backgrounds around the position x^m of the D3-branes

$$
\begin{aligned}
Z_1^{-1/2} &= 1 + \frac{1}{2} K_{mn}\, x^m x^n + \ldots \\
Z_2^{1/2} &= 1 + \ldots \\
\tau &= \tau_0 + \frac{1}{2} \tau_{mn}\, x^m x^n \\
\chi_4 &= (\text{const.} + \frac{1}{2} \chi_{mn}\, x^m x^n + \ldots)\, dx^0 dx^1 dx^2 dx^3 \\
G_{lmn}(x^m) &= G_{lmn} + \ldots
\end{aligned} \tag{2.2}
$$

where the coefficients K, F, G, τ in the right hand side are constant, independent of x^m. The piece of the 5-form background relevant for our purposes below is

$$
F_5 = \frac{1}{2}(\chi_{mn} + \chi_{nm}) x^m dx^n dx^0 dx^1 dx^2 dx^3 + \ldots \tag{2.3}
$$

In the absence of fluxes, the massless fields on a stack of n D3-branes are given by $U(n)$ gauge bosons, six real adjoint scalars, and four Majorana adjoint fermions, the latter transforming in the representations $\mathbf{6}$ and $\overline{\mathbf{4}}$ of the $SO(6)$ local symmetry. In $N = 1$ supersymmetry language, they fill out a vector multiplet and three chiral multiplets. The effect of the above backgrounds on the effective action of the worldvolume fields can be obtained by plugging the above expansions in the DBI and CS action for the D3-branes.

Calabi-Yau manifolds are endowed with a complex structure, hence it is useful to rewrite the world-volume fields and the 3-form background in complex coordinates. Let us introduce local complex coordinates $z^1 = \frac{1}{\sqrt{2}}(x^4 + ix^5)$, $z^2 = \frac{1}{\sqrt{2}}(x^6 + ix^7)$, $z^3 = \frac{1}{\sqrt{2}}(x^8 + ix^9)$. We define the complex scalars $\Phi^1 = \frac{1}{\sqrt{2}}(\phi^4 + i\phi^5)$, $\Phi^2 = \frac{1}{\sqrt{2}}(\phi^6 + i\phi^7)$, $\Phi^3 = \frac{1}{\sqrt{2}}(\phi^8 + i\phi^9)$, and use $\Phi^{\bar{i}}$ to denote $(\Phi^i)^*$. Denoting the four-plet of fermions by their $SO(6)$ weights, the fermion $\frac{1}{2}(+ + +)$ belongs to the $N = 1$ vector multiplet, and is referred to as the gaugino, denoted λ. The fermions combining with the above

complex scalars to give $N=1$ chiral supermultiplets are Ψ^1, Ψ^2, Ψ^3, corresponding to the weights $\frac{1}{2}(+--)$, $\frac{1}{2}(-+-)$ and $\frac{1}{2}(--+)$, respectively.

In a preferred complex structure, only an $SU(3)$ (times $U(1)$) subgroup of the $SO(6)$ symmetry is manifest, hence it is useful to decompose the 3-form flux background under it. The antisymmetric flux G_{mnp} transforms as a 20-dimensional reducible $SO(6)$ representation, decomposing as $\mathbf{20}=\overline{\mathbf{10}}+\mathbf{10}$. The irreducible representations $\overline{\mathbf{10}}$, $\mathbf{10}$ correspond to the imaginary self-dual (ISD) $G_{(3)}^{+}$ and imaginary anti self-dual (IASD) $G_{(3)}^{-}$ parts, respectively, defined as

$$G_{(3)}^{\pm} \;=\; \frac{1}{2}(G_{(3)}\mp i *_6 G_{(3)}) \;\;;\;\; *_6 G_{(3)}^{\pm} \;=\; \pm i G_{(3)}^{\pm} \tag{2.4}$$

It is useful to classify the components of the ISD and IASD parts of G_3 according to their behaviour under $SU(3)$, $\mathbf{10}=\mathbf{6}+\mathbf{3}+\mathbf{1}$. For that purpose, we introduce the tensors [26]

$$\begin{aligned} S_{ij} &= \frac{1}{2}(\epsilon_{ikl}G_{j\bar{k}\bar{l}}+\epsilon_{jkl}G_{i\bar{k}\bar{l}}) \\ A_{\bar{i}\bar{j}} &= \frac{1}{2}(\epsilon_{\bar{i}\bar{k}\bar{l}}G_{kl\bar{j}}-\epsilon_{\bar{j}\bar{k}\bar{l}}G_{kl\bar{i}}) \end{aligned}$$

defined in terms of the complex components of G_3, and which transform in the representation $\mathbf{6}$ and $\mathbf{3}$ under $SU(3)$. One similarly defines $S_{\bar{i}\bar{j}}$ and A_{ij}. The $SU(3)$ properties of each component is displayed in table 1.

	ISD			IASD	
$SU(3)$ rep.	Form	Tensor	$SU(3)$ rep.	Form	Tensor
$\overline{1}$	$(0,3)$	$G_{\bar{1}\bar{2}\bar{3}}$	1	$(3,0)$	G_{123}
$\overline{6}$	$(2,1)_P$	$S_{\bar{i}\bar{j}}$	6	$(1,2)_P$	S_{ij}
$\overline{3}$	$(1,2)_{NP}$	A_{ij}	3	$(2,1)_{NP}$	$A_{\bar{i}\bar{j}}$

Table 1: $SU(3)$ decomposition of antisymmetric $G_{(3)}$ fluxes.

In [18] the soft term Lagrangian for the worldvolume fields of D3-branes was computed to be

$$\begin{aligned} \mathcal{L} = \mathrm{Tr}\Big[& -(\,2K_{i\bar{j}}-\chi_{i\bar{j}}+g_s(\mathrm{Im}\,\tau)_{i\bar{j}}\,)\,\Phi^i\Phi^{\bar{j}} \;-\; \tfrac{1}{2}\,(\,2K_{ij}-\chi_{ij}+g_s(\mathrm{Im}\,\tau)_{ij}\,)\,\Phi^i\Phi^j \;+\;\mathrm{h.c.} \\ & +g_s\sqrt{2\pi}\,\Big[\,\tfrac{1}{3}G_{123}\,\epsilon_{ijk}\,\Phi^i\Phi^j\Phi^k+\tfrac{1}{2}\epsilon_{\bar{i}\bar{j}\bar{l}}\,(S_{lk}-(A_{\bar{l}\bar{k}})^*)\,\Phi^{\bar{i}}\Phi^{\bar{j}}\Phi^k\;+\;\mathrm{h.c.}\Big]\;+ \\ & +\tfrac{g_s^{1/2}}{2\sqrt{2}}\,\Big[\,G_{123}\,\lambda\lambda\;+\;\tfrac{1}{2}\epsilon_{ijk}A_{\bar{j}\bar{k}}\,\Psi^i\lambda\;+\;\tfrac{1}{2}S_{ij}\,\Psi^i\Psi^j\;+\;\mathrm{h.c.}\Big]\,\Big] \end{aligned} \tag{2.5}$$

where we have defined

$$(\mathrm{Im}\,\tau)_{i\bar{j}}=\frac{1}{2i}(\tau_{i\bar{j}}-(\tau_{j\bar{i}})^*) \;\;;\;\; (\mathrm{Im}\,\tau)_{ij}=\frac{1}{2i}(\tau_{ij}-(\tau_{\bar{j}\bar{i}})^*) \tag{2.6}$$

In the notation of the first appendix in [18] one thus has soft terms for the D3-brane worldvolume fields

$$
\begin{aligned}
m^2_{i\bar{j}} &= 2K_{i\bar{j}} - \chi_{i\bar{j}} + g_s(\mathrm{Im}\,\tau)_{i\bar{j}} \\
B_{ij} &= 2K_{ij} - \chi_{ij} + g_s(\mathrm{Im}\,\tau)_{ij} \\
A^{ijk} &= -h^{ijk}\frac{g_s^{1/2}}{\sqrt{2}}\,G_{123} \\
C^{ijk} &= +h^{ijl}\frac{g_s^{1/2}}{2\sqrt{2}}(S_{lk} - (A_{\overline{lk}})^*) \\
M^a &= \frac{g_s^{1/2}}{\sqrt{2}}\,G_{123} \\
\mu_{ij} &= -\frac{g_s^{1/2}}{2\sqrt{2}}S_{ij} \\
M_g^{ia} &= \frac{g_s^{1/2}}{4\sqrt{2}}\epsilon_{ijk}A_{\bar{j}\bar{k}}
\end{aligned}
\tag{2.7}
$$

The supergravity equations of motion impose some constraints on the background, namely [18]

$$
4\sum K_{l\bar{l}} = \frac{g_s}{2}\left(|G_{123}|^2 + |G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}\sum_{ij}(|S_{ij}|^2 + |S_{\overline{ij}}|^2 + |A_{ij}|^2 + |A_{\overline{ij}}|^2)\right) \tag{2.8}
$$

$$
-2\sum \chi_{l\bar{l}} = \frac{g_s}{2}\left(|G_{123}|^2 - |G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}\sum_{ij}(|S_{ij}|^2 - |S_{\overline{ij}}|^2 - |A_{ij}|^2 + |A_{\overline{ij}}|^2)\right) \tag{2.9}
$$

$$
i\sum \tau_{l\bar{l}} = \frac{1}{2}\left(G_{123}G_{\bar{1}\bar{2}\bar{3}} + \frac{1}{4}S_{lk}S_{\overline{lk}} + \frac{1}{4}A_{lk}A_{\overline{lk}}\right) \tag{2.10}
$$

They allow to relate the scalar masses to the flux background, as follows

$$
\begin{aligned}
m_1^2 + m_2^2 + m_3^2 = \frac{g_s}{2}\Big[& |G_{123}|^2 + \frac{1}{4}\sum_{ij}(|S_{ij}|^2 + |A_{\overline{ij}}|^2) - \\
& - \mathrm{Re}\,(G_{123}G_{\bar{1}\bar{2}\bar{3}} + \frac{1}{4}S_{lk}S_{\overline{lk}} + \frac{1}{4}A_{lk}A_{\overline{lk}})\Big]
\end{aligned}
\tag{2.11}
$$

As emphasized in [18], the scalar mass matrix is not fully determined in terms of the fluxes, only its trace is.

Our purpose in the present paper is to carry out a similar analysis for the effect of fluxes on configurations including D7-branes.

3 D7-branes and RR, NSNS 3-form fluxes

3.1 D7-branes and the 3-form flux background

There are a number of differences between the case of D7 and that of D3-branes reviewed in the previous section. First of all the D7-branes in general wrap a 4-cycle Σ_4 on the internal space. Thus an expansion of the closed string background on all 6 transverse coordinates x^m as we did in the D3-brane case is no longer the correct procedure. Rather, we need to describe the local geometry around the 4-cycle wrapped by the D7-brane, namely a tubular neighbourhood around the 4-cycle, given by the normal bundle of the 4-cycle, i.e. the wrapped 4-cycle, on which we fiber the normal direction. Then we may expand the background in a power series in its dependence on the normal coordinate (while in principle keeping its full dependence on Σ_4). For most purposes, we will center on cases where the fibration is trivial, and the local geometry is $\Sigma_4 \times C$. [3] We will comment later on possible generalizations of this background.

The strategy then is to obtain the 8d action for the D7-brane, taking the effect of fluxes into account. An important ingredient is that the symmetry of the problem is reduced as compared with the D3-brane case. In fact, the supergravity background need only respect 4d Lorentz invariance. On the other hand, the local geometric symmetry in the internal space is only $SO(4) \times SO(2)$, where the first factor is the local euclidean rotation group on Σ_4, and the second in the normal direction. Hence the local 8d flux-induced terms must respect that symmetry (taking into account its action on the world-volume fields as well).

A second step involves the Kaluza-Klein compactification of this 8d action on Σ_4 to 4d. In this process, the geometry of Σ_4 plays a crucial role even in the absence of fluxes, and determines the number and kind of the massless fields on the D7-brane gauge sector in 4d. Our purpose is to determine the effect of the 8d flux induced terms on these massless fields, hence to compute it we will need to center on concrete cases. Our analysis will mainly center on the case of T^4 (which can be subsequently employed to study orbifolds thereof), and K3, with trivial worldvolume gauge bundles.

We are thus interested in determining first the 8d action, including the effects of fluxes, and second in discussing the compactification to 4d. We will concentrate on the most relevant terms, namely up to dimension three, hence scalar masses, scalar

[3] This restricts us to backgrounds $T^4 \times C$ or K3$\times C$, which is a restricted set, but sufficiently rich to have non-trivial effects. Moreover, the qualitative features can be extrapolated to more involved situations, see section 8.

trilinears and fermion masses.

Still this is a rather involved problem in the general case, and we will have to make some restricting assumptions on the geometry. In particular, we assume certain restricted classes of closed string backgrounds. The bottomline is that we consider the flux to have the structure in eq. (3.6), and to be pure ISD or pure IASD; we will also center in situations where the background is constant over Σ_4.

To specify our assumptions and also for practical purposes it is now useful to write the 3-form fluxes G_{mnp} in terms of the local geometric symmetry $SO(4) \times SO(2)$. Thus now the decomposition of the ISD (IASD) pieces of G_3 in representations of the $SU(2) \times SU(2)' \times U(1)$ symmetry are as follows

$$10 \;=\; (3,1)_- + (1,3')_+ + (2,2')_0 \quad , \quad \overline{10} \;=\; (3,1)_+ + (1,3')_- + (2,2')_0 \tag{3.1}$$

where the subindex gives the ± 1 $U(1)$ charge. Choosing our 4-cycle to be parametrized by z^1, z^2, and localized in the transverse direction z^3, the triplets of $SU(2), SU(2)'$ are related with the fluxes in $SU(3)$ notation by

$$\begin{aligned}
(1,3')_+ &= \{\mathcal{G}'_0 = -\frac{1}{\sqrt{2}}A_{\bar{1}\bar{2}} \ , \ \mathcal{G}'_x = -\frac{1}{\sqrt{2}}(\frac{1}{2}S_{33} - G_{123}) \ , \ \mathcal{G}'_y = -\frac{i}{\sqrt{2}}(\frac{1}{2}S_{33} + G_{123})\} \\
(3,1)_- &= \{\mathcal{G}_0 = \frac{1}{\sqrt{2}}S_{12} \ , \ \mathcal{G}_x = -\frac{1}{\sqrt{2}}(\frac{1}{2}S_{11} - \frac{1}{2}S_{22}) \ , \ \mathcal{G}_y = -\frac{i}{\sqrt{2}}(\frac{1}{2}S_{11} + \frac{1}{2}S_{22})\} \\
(1,3')_- &= \{G'_0 = -\frac{1}{\sqrt{2}}A_{12} \ , \ G'_x = -\frac{1}{\sqrt{2}}(\frac{1}{2}S_{\bar{3}\bar{3}} - G_{\bar{1}\bar{2}\bar{3}}) \ , \ G'_y = -\frac{i}{\sqrt{2}}(\frac{1}{2}S_{\bar{3}\bar{3}} + G_{\bar{1}\bar{2}\bar{3}})\} \\
(3,1)_+ &= \{G_0 = \frac{1}{\sqrt{2}}S_{\bar{1}\bar{2}} \ , \ G_x = -\frac{1}{\sqrt{2}}(\frac{1}{2}S_{\bar{1}\bar{1}} - \frac{1}{2}S_{\bar{2}\bar{2}}) \ , \ G_y = -\frac{i}{2}(\frac{1}{2}S_{\bar{1}\bar{1}} + \frac{1}{2}S_{\bar{2}\bar{2}})\}
\end{aligned} \tag{3.2}$$

Regarding the triplets of $SU(2)$ groups as vectors of $SO(3)$, for future convenience we define the $SU(2)$ invariant scalar product

$$A \cdot B = A_0 B_0 + A_x B_x + A_y B_y \tag{3.3}$$

On the other hand the G_{mnp} components transforming like $(2,2)_0$ correspond to the SU(3) components $S_{i3}, A_{i3}, S_{\bar{i}\bar{3}}, A_{\bar{i}\bar{3}}, i = 1, 2$. These fluxes are special in several respects. In particular, if Σ_4 contains 3-cycles C_3 , this multiplet contains fluxes such that

$$\int_{C_3} F_3 \neq 0 \quad ; \quad \int_{C_3} H_3 \neq 0 \tag{3.4}$$

This is the case for instance for T^4, on which much of our analysis centers. This is problematic because non-zero integrals of H_3 on a D-brane cycle generate a world-volume tadpole for the dual world-volume gauge potential $\int_{D7} H_3 \wedge A_5$, rendering the

configuration inconsistent [27] [4]. Moreover, in general, such fluxes along world-volume directions are quantized, and cannot be diluted away keeping the D7-brane physics four-dimensional. Hence their presence can lead to qualitatively large changes in the 4d physics, which may not be well described with our techniques (which are implicitly perturbative in the flux density).

For these reasons, we restrict our analysis to situations where fluxes in the $(2,2')_0$ multiplet are absent. Still, the remaining fluxes transforming like $(3,1)+(1,3')$ include the most interesting cases, and lead to interesting and non-trivial effects.

For the computation of soft terms up to the order of interest, it is enough to consider the leading term in the expansion of G_3 in the normal direction, namely the z_3-independent term. Assuming in addition that the background 3-form flux is independent of the coordinates on the 4-cycle, as mentioned above, one may obtain the NSNS and RR 2-form potentials, in a particular gauge [5]

$$\begin{aligned} B_{mn} &= \frac{g_s}{6i} G^*_{mnp} z^p - \frac{g_s}{12i}\epsilon_{mnq} S_{\bar{q}\bar{p}} \bar{z}^p + \frac{g_s}{12i}\epsilon_{mnq} A^*_{\bar{q}\bar{p}} \bar{z}^p - \\ &\quad - \frac{g_s}{6i} G_{mnp} z^p + \frac{g_s}{12i}\epsilon_{mnq} S^*_{\bar{q}\bar{p}} \bar{z}^p - \frac{g_s}{12i}\epsilon_{mnq} A_{\bar{q}\bar{p}} \bar{z}^p \\ B_{\bar{m}\bar{n}} &= -\frac{g_s}{6i} G_{\bar{m}\bar{n}\bar{p}} \bar{z}^p + \frac{g_s}{12i}\epsilon_{\bar{m}\bar{n}\bar{q}} S^*_{qp} z^p - \frac{g_s}{12i}\epsilon_{\bar{m}\bar{n}\bar{q}} A_{qp} z^p + \\ &\quad + \frac{g_s}{6i} G^*_{\bar{m}\bar{n}\bar{p}} \bar{z}^p - \frac{g_s}{12i}\epsilon_{\bar{m}\bar{n}\bar{q}} S_{qp} z^p + \frac{g_s}{12i}\epsilon_{\bar{m}\bar{n}\bar{q}} A^*_{qp} z^p \\ B_{m\bar{n}} = -B_{\bar{n}m} &= \frac{g_s}{12i}\epsilon_{mpq} S_{\bar{q}\bar{n}} z^p + \frac{g_s}{12i}\epsilon_{\bar{n}\bar{p}\bar{q}} S^*_{qm} \bar{z}^p - \frac{g_s}{12i}\epsilon_{\bar{n}\bar{p}\bar{q}} A_{qm} \bar{z}^p - \frac{g_s}{12i}\epsilon_{mpq} A^*_{\bar{q}\bar{n}} z^p - \\ &\quad - \frac{g_s}{12i}\epsilon_{\bar{n}\bar{p}\bar{q}} S_{qm} \bar{z}^p - \frac{g_s}{12i}\epsilon_{mpq} S^*_{\bar{q}\bar{n}} z^p + \frac{g_s}{12i}\epsilon_{mpq} A_{\bar{q}\bar{n}} z^p + \frac{g_s}{12i}\epsilon_{\bar{n}\bar{p}\bar{q}} A^*_{qm} \bar{z}^p \end{aligned} \tag{3.5}$$

The RR 2-forms may be obtained analogously, but we will not need its explicit expression.

Out of the above components, only those linear in z^3, $\bar{z}^3$ will actually be relevant in the computation of soft terms, see appendix A. This suggests that the computation of the 2-form gauge potential can be recast in a more compact way [6], as follows. The $(1,3')+(3,1)$ fluxes have always two legs in Σ_4. This allows us to associate to each of the above $SO(4)\times SO(2)$ representations a different 2-form in Σ_4. In fact, it is possible

[4]This can be solved as for the baryonic brane in [28], by introducing additional branes ending on the D7-branes [29], but this completely changes the kind of configurations here considered.

[5]We have chosen a gauge on which all the coordinates are on equal footing. This is somehow a natural choice for a closed string background, which is independent of the D-brane configuration under consideration.

[6]An additional advantage is that, as we argue in section 8, this derivation is valid even in situations with 3-form backgrounds non-constant over the 4-cycle.

to decompose G_3 as

$$G_3 = \beta \wedge dz^3 + \beta' \wedge d\bar{z}^3 + \gamma \wedge dz^3 + \gamma' \wedge d\bar{z}^3 \tag{3.6}$$

where β, β' and γ, γ' are selfdual and anti selfdual two-forms in Σ_4, namely

$$*_4\beta = \beta \quad ; \quad *_4\gamma = -\gamma \tag{3.7}$$

(and similarly for the primed forms), corresponding respectively to the $(1,3')_+$, $(1,3')_-$, $(3,1)_+$ and $(3,1)_-$ pieces of G_3.

For clarity we summarize in table 3.1 the main properties of these induced 2-forms. The above scalar product between SU(2) triplets will induce a positive definite product

Form	SD/ASD in Σ_4	Corresponding G_3 rep.	ISD/IASD flux
β	SD	$(1,3)_+$	IASD
β'	SD	$(1,3)_-$	ISD
γ	ASD	$(3,1)_+$	ISD
γ'	ASD	$(3,1)_-$	IASD

Table 2: Properties of the 2-forms induced by the flux in Σ_4.

in Σ_4 given by

$$\omega_1 \cdot \omega_2 = \int_{\Sigma_4} \omega_1 \wedge *_{\Sigma_4}\omega_2 \tag{3.8}$$

With all of this, the part of the B-field laying completely in Σ_4 is given by

$$B_2|_{\Sigma_4} = -\frac{g_s}{6i}\left((\beta-\beta^*)z^3 + (\beta'-\beta'^{\,*})z^{\bar{3}} + (\gamma-\gamma^*)z^3 + (\gamma'-\gamma'^{\,*})z^{\bar{3}}\right) \tag{3.9}$$

with $(\beta^*)_{mn} = (\beta'_{\bar{m}\bar{n}})^*$, etc.

In particular, it is important to notice the explicit dependence of these components of the B-field on the transverse coordinates. As mentioned, these are the only components relevant in the computation of soft terms (see appendix A).

We will further assume that the flux background is purely imaginary self-dual (ISD) or purely imaginary anti-selfdual (IASD). This assumption is not compulsory, but simplifies enormously the computations, since in this situation the equations of motion imply that the dilaton is constant. It is important to mention that consequently, although our expressions below include both ISD and IASD components, it is understood that they are valid just when only ISD or only IASD components are turned on. Namely, the expressions below do not contain possible interference terms that may arise when both are present.

Concerning the rest of the closed string backgrounds, we will consider a general form analogous to that considered above for D3-branes

$$\begin{aligned} ds^2 &= Z_1^{-1/2}(x)\eta_{\mu\nu}dx^\mu dx^\nu + Z_2(x)^{1/2}dx^m dx^m \\ \chi_4 &= \chi(x)dx^0dx^1dx^2dx^3 \quad ; \quad F_5 = d\chi_4 + *_{10}d\chi_4 \end{aligned} \tag{3.10}$$

Expanding on $z^3, \bar{z}^3$ the warp factor and 5-form flux, we have

$$\begin{aligned} Z_1^{-1/2}(x) &= 1 + \frac{1}{2}K_{mn}(x^a)\; x^m x^n + \dots \\ Z_2^{1/2}(x) &= 1 + \frac{1}{2}K'_{mn}(x^a)\; x^m x^n + \dots \\ \chi_4(x) &= \frac{1}{2}\chi_{mn}(x^a)\; x^m x^n + \dots \end{aligned} \tag{3.11}$$

with $m, n = 8, 9$. At this level, K, K' and χ may depend on the longitudinal components $z^{1,2}$, $\bar{z}^{1,2}$. The conditions these backgrounds must satisfy in order to solve Type IIB supergravity equations of motion are discussed in section 3.4.

As a last comment, notice that the metric ansatz does not contain any component mixing the z_3 coordinate with the z_1, z_2. Hence we take the local geometry to factorize as $\Sigma_4 \times C$, namely we work on local geometries where the normal bundle is trivial. This restricts us to $T^4 \times C$ and K3$\times C$, although clearly the general technique may be applied to non-trivial normal bundles.

3.2 The $D = 8$ action

Our starting point will be the $D = 8$ DBI and CS actions for the D7-brane. Myers' action in the Einstein's frame for a D7-brane is given by [7]

$$\begin{aligned} S = \;& -\mu_7 \int d^8\xi \, STr \left[e^{-\phi} \sqrt{-\det(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] \,+\, \sigma F_{\mu\nu})\, \det(Q_{ij})} \right] \\ & + \mu_7 g_s \int STr \left(P\left[\sigma C_6 F_2 + C_8 - C_6 B_2\right]\right) \end{aligned}$$

where i, j run over the six directions transverse to 4d Minkowski space, $P[M]$ denotes the pullback of the 10d background onto the D7-brane worldvolume, and

$$\begin{aligned} E_{MN} &= g_s^{1/2} G_{MN} - B_{MN} \\ Q^m{}_n &= \delta^m{}_n + i\sigma\,[\phi^m, \phi^p]\, E_{pn} \\ \sigma &= 2\pi\alpha' \end{aligned} \tag{3.12}$$

[7] In the CS action we only keep pieces giving possible contributions to the soft terms.

We will label by $a, b = 1, \bar{1}, 2, \bar{2}$ and $m, n = 3, \bar{3}$ the longitudinal and transverse indices to the D7-brane worldvolume. Within the class of backgrounds considered, we have $g_{am} = 0$, thus

$$P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^{ij} E_{j\nu}] = P[g_{\mu\nu} g_s^{1/2} - B_{ab}\delta^a_\mu \delta^b_\nu + \delta^a_\mu B_{am}(Q^{-1} - \delta)^{mn} B_{nb}\delta^b_\nu] \quad (3.13)$$

where Q is given by

$$Q^i{}_j = \delta^i{}_j + i\sigma[\Phi^i, \Phi^k](G_{kj} g_s^{1/2} - B_{kj}) \quad (3.14)$$

Expanding the above determinants we arrive to the 8d action in terms of the NS and RR background. The details of the computation can be found in the appendix A. The contributions to the soft terms are given by

$$\begin{aligned} S_{soft} = \mu_7 g_s STr \int \Big[& (Z_1^{-1}(\Phi^3, \Phi^{\bar{3}}) Z_2(\Phi^3, \Phi^{\bar{3}}) dvol_{4d} \wedge dz^1 \wedge d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^2 - \\ & -\frac{1}{2}(B_2|_{\Sigma_4} - \sigma F_2) \wedge *_4(B_2|_{\Sigma_4} - \sigma F_2) dvol_{4d} + \\ & +(C_8 - C_6 \wedge B_2 + \sigma C_6 \wedge F_2)|_{M^4 \times \Sigma_4} \Big] + \ldots \end{aligned} \quad (3.15)$$

with $B_2|_{\Sigma_4}$ given in eq. (3.9). Here the dots refer to derivative couplings and other 8-dimensional contributions which are not explicitly needed in our computations below.

Writing now C_8 and C_6 in terms of the two forms induced in Σ_4 by the flux, and plugging them, together with (3.9), into the above expression we obtain the eight dimensional soft lagrangian. We refer again to the appendix A for details. The final result for the $D = 8$ bosonic action in terms of $SU(3)$ flux components, including the kinetic and quartic terms is [8]

$$\begin{aligned} \mathcal{L} \;=\; & \mu_7\, g_s\, \sigma^2\, \mathrm{STr}\, Z_1^{-1}(\Phi^3, \Phi^{\bar{3}})\, Z_2(\Phi^3, \Phi^{\bar{3}}) \left[\frac{1}{\sigma^2} + \partial_\mu \Phi^3 \partial_\mu \Phi^{\bar{3}} - \right. \\ - \; & \frac{g_s}{18} \left(\frac{1}{4}(S_{12})^2 + \frac{1}{4}(A_{12})^2 - \frac{1}{2} G_{\bar{1}\bar{2}\bar{3}} S_{\bar{3}\bar{3}} - \frac{1}{4} S_{22} S_{11} \right) \Phi^{\bar{3}} \Phi^{\bar{3}} + h.c. - \\ - \; & \frac{g_s}{18} \left[|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}|S_{\bar{3}\bar{3}}|^2 + \frac{1}{4} \sum_{i,j=1,2} (|S_{ij}|^2 + |A_{ij}|^2) \right] \Phi^{\bar{3}} \Phi^3 - \\ + \; & \sum_{j,k,p=1,2} \frac{g_s}{3} \epsilon_{3jk} \, [\, (S_{kp})^* + (A_{kp})^*] \; \Phi^3 A^{[j} A^{\bar{p}]} + h.c. + \\ - \; & \frac{g_s}{6} \epsilon_{ij3}\, S_{\bar{3}\bar{3}} A^i A^j \Phi^{\bar{3}} + h.c. - \frac{2g_s}{3}\, G_{\bar{1}\bar{2}\bar{3}} \Phi^{\bar{3}} A^{[\bar{1}} A^{\bar{2}]} + h.c. - \\ - \; & \left. g_s [A^a, A^{(b}][A^{\bar{b})}, A^{\bar{a}}] + \partial_\mu A^a \partial_\mu A^{\bar{a}} - \frac{1}{2} g_s ([\Phi^3, \Phi^{\bar{3}}])^2 - \frac{1}{2} g_s [A^a, \Phi^3][\Phi^{\bar{3}}, A^{\bar{a}}] \;\; \right] + \;\ldots \end{aligned} \quad (3.16)$$

[8]Notice that the derivative terms mentioned below are required to make the expression local Lorentz invariant in 8d. However, the listed terms are the only ones explicitly need for our discussion below.

where A^a are the components of the 8-dimensional gauge boson along the directions longitudinal to the D7-brane and Φ^3 are (adjoint) scalar fields describing the transverse degrees of freedom of the D7-brane. To avoid long expressions we have not included here either the YM action of $D = 4$ gauge bosons or couplings depending on derivatives of the dimensions longitudinal to the D7-brane. Those terms are understood to be included in the dots, but, as we commented before, they will not be relevant for our discussion below.

The explicit dependence on the antisymmetric backgrounds (in $SU(3)$ notation) and the warp factor is shown. Concerning the latter, notice that for pure ISD or pure IASD fluxes, the equations of motion require the warp factor to correspond to a black 3-brane solution, with $Z_1 = Z_2$, and the warp factor dependence drops from the above expression [9]. We will consider this situation in what follows.

Some of the terms in (3.17) correspond to the Yang-Mills action for the $D = 8$ gauge boson and the kinetic terms for the scalars Φ^3. In addition there are terms which depend on the 3-form fluxes. In particular note that the transverse scalars Φ^3 get mass terms for some non-vanishing fluxes. This is an important point since it means that, irrespective of the compactification space, in general 3-form fluxes stabilize the positions of the D7-branes. This was already discussed in section 5 in [18]. Note also that in addition there are SUSY-breaking scalar-(vector)^2 couplings proportional to some of the fluxes. Upon further compactification to $D = 4$ these terms will give rise to trilinear scalar couplings, as described below.

It may be useful for later purposes to display this result in terms of the $SU(2) \times SU(2)' \times U(1)$ flux components

$$\begin{aligned}
\mathcal{L} \;=\;& \mu_7\, g_s\, \sigma^2 \mathrm{STr}\left[\frac{1}{\sigma^2} + \partial_\mu \Phi^3 \partial_\mu \Phi^{\bar{3}} - \right. \\
&- \frac{g_s}{36}\left[\mathcal{G}^* \cdot \mathcal{G}^* + (G')^* \cdot (G')^*\right] \Phi^3\Phi^3 + h.c. - \frac{g_s}{18}\left(\mathcal{G}\cdot\mathcal{G}^* + G'\cdot(G')^*\right)\Phi^{\bar{3}}\Phi^3 - \\
&- \frac{g_s\sqrt{2}}{3}\Phi^3\left[\left(A^{\bar{1}} \quad A^{\bar{2}} \right)(\mathcal{G}^*\cdot\vec{\sigma})\begin{pmatrix} A^1 \\ A^2 \end{pmatrix} + \left(A^{\bar{1}} \quad A^{2} \right)(G'^*\cdot\vec{\sigma})\begin{pmatrix} A^1 \\ A^{\bar{2}} \end{pmatrix} + h.c. \right] - \\
&- \left. g_s\,[A^a, A^{(b}][A^{\bar{b})}, A^{\bar{a}}] + \partial_\mu A^a \partial_\mu A^{\bar{a}} - \frac{1}{2} g_s([\Phi^3,\Phi^{\bar{3}}])^2 - \frac{1}{2} g_s [A^a,\Phi^3][\Phi^{\bar{3}}, A^{\bar{a}}] \;\right]
\end{aligned} \tag{3.17}$$

where $\vec{\sigma}$ is the vector of Pauli matrices given by

$$\vec{\sigma} = \{\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\} \tag{3.18}$$

[9] This is similar to the cancellation of force in systems of a D7-brane in the background of D3- or anti-D3-branes, whose supergravity solution is also of the black 3-brane form.

Note the important fact that the effective action depends on the backgrounds of type $\mathcal{G}$ and G', but not on the backgrounds of type $\mathcal{G}'$ and G. In terms of the local R-symmetries $SU(2) \times SU(2)' \times U(1)$ this means that only backgrounds with negative $U(1)$ charge (i.e. $(3,1)_- + (1,3)_-$) appear in the effective action. In other words, only fluxes with a particular correlation between their self-duality properties in the threefold and their duality properties on the 4-cycle give rise to soft terms, while fluxes with the opposite correlation lead to cancellation between the DBI and the CS contributions.

This suggests the following physical interpretation. Consider for instance fluxes in the $(1,3')_-$ representation. These are ISD in the threefold, and hence are similar to a D3-brane distribution (rather than a $\overline{D3}$-brane one). On the other hand, they correspond to a selfdual NSNS 2-form on the D7-brane volume, which induces a $\overline{D3}$-brane charge on the latter [10]. The non-cancellation of the interaction between the induced $\overline{D3}$-brane on the D7-brane with the effective D3-brane distribution in the bulk leads to non-trivial soft terms. A similar argument may be applied for fluxes in the $(3,1)_-$, which are IASD in the threefold, and anti-selfdual on the 4-cycle. Finally for fluxes in the $(3,1)_+$ or $(1,3')_+$ the induced charges in the threefold and the D7-brane volume are of the same kind, leading to cancellations between DBI and CS contributions.

One should in principle also compute the 8d action for the world-volume fermion fields. However, it will be simpler to do it directly in terms of the 4d action, exploiting the symmetries of the system, as we do below.

3.3 The $D = 4$ action and soft terms

3.3.1 Generalities

Up to now our discussion has been quite general, and independent of the 4-cycle wrapped by the D7-brane (although as discussed, certain 8d derivative terms may be necessary to properly describe some situations). To proceed with a general Σ_4 we would have to make a Kaluza-Klein reduction and compute the massless spectrum, which is very model dependent.

In this process, even in the absence of fluxes, the non-trivial geometry determines the number of massless 4d scalars as the number of zero modes of 8d scalars and internal components of the gauge fields. A prototypical example is that the number of Wilson line moduli is given by the number of harmonic 1-forms on Σ_4 [11]. This is

[10] Notice that our convention at this point differs from the usual one.

[11] If the D7-branes carry a non-trivial gauge bundle on Σ_4, the counting of bundle moduli is more

very often vanishing, but is four for T^4. In addition, the 8d complex scalar fields has a number of zero modes given by the the number of independent sections of the normal bundle (counted by $h^{2,0}(\Sigma_4)$). This may be zero in particular cases of rigid 4-cycles (like D7-branes on the exceptional P_2 in the blow-up of the $\mathbf{Z_3}$ orbifold singularity $\mathcal{O}_{-3}(\mathbf{P_2})$, or on complex delPezzo surfaces). In other situations there may be multiple massless scalars, associated to different zero modes of the 8d scalar. One example is to consider a compactification K3$\times T^2$, and the D7-branes wrapped on $\Sigma_4 = C_g \times T^2$, with C_g a holomorphic genus g Riemann surface in K3. The number of 4d scalars is given by the dimension of the moduli space of holomorphic genus g Riemann surfaces in K3, given by g.

In cases where the normal bundle is trivial, like $T^4 \times C$ or K3$\times C$, there is one massless complex scalar Φ_3. It corresponds to a zero mode of the 8d massless scalars, with constant profile on Σ_4. This will be important for our discussion below.

Any mode which is made massive by the compactification has a typical mass scale of $1/R$. This is, in the large volume regime of validity of the description, much larger than the typical flux-induced mass scales, of order α'/R^3. Hence we are interested in describing the effect of fluxes on the light modes of the configuration, which are the relevant degrees of freedom from the 4d perspective. There are two sources for these effects. First, the presence of the fluxes introduces new 8d derivative couplings of the 8d fields; these would enter into the kinetic energy operator of the internal space for these fields, and could induce e.g. non-trivial masses for their former zero modes. Second, there are 8d non-derivative terms, like 8d mass terms for the 8d Φ^3 scalar, which certainly modify the effective action of the corresponding 4d zero modes, leading to non-trivial soft terms. Full determination of these effects however requires a precise knowledge of the internal profile of these zero modes, and hence a particular choice of 4-cycle.

In this section we center on D7-branes on T^4. A similar discussion for D7-branes on K3 is carried out in section 8.1.

3.3.2 The T^4 case: bosonic sector

In this section we center on the particularly simple case of D7-branes wrapped on T^4 on $T^4 \times C$, where we moreover assume the background to be independent of the 4-cycle coordinates (another interesting and tractable case, namely Σ_4 =K3 is described in section 8.1). This may be also trivially generalized to the orbifold $(T^4 \times C)/Z_N$ case,

involved. However, we will not consider this situation.

which can lead to chiral fermions and semirealistic models, see section 9.

In the toroidal case the 4d massless modes have constant profile in the internal 4-cycle. The final spectrum in the absence of fluxes corresponds to an $N = 4$ theory. The $D = 8$ gauge bosons A^a give rise to two more complex adjoint scalars $\Phi^{1,2}$, associated to Wilson line degrees of freedom.

Integrating eq.(3.17) over the four toroidal dimensions it is easy to obtain the $D = 4$ bosonic action

$$\begin{aligned}
\mathcal{L} &= \mathrm{Tr}\,\{\,\partial_\mu \Phi^m \partial_\mu \Phi^{\bar{m}} - \qquad (3.19)\\
&- \frac{g_s}{18}\left(\frac{1}{4}[(S_{12})^*]^2 + \frac{1}{4}[(A_{12})^*]^2 - \frac{1}{2}(G_{\bar{1}\bar{2}\bar{3}})^*(S_{\bar{3}\bar{3}})^* - \frac{1}{4}(S_{22})^*(S_{11})^*\right)\Phi^3\Phi^3 + h.c.\\
&- \frac{g_s}{18}\left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}|S_{\bar{3}\bar{3}}|^2 + \frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2 + |A_{ij}|^2)\right)\Phi^{\bar{3}}\Phi^3 +\\
&+ \sum_{k,p=1,2}\frac{g_s^{1/2}g_{YM}}{6}\epsilon_{ijk}((S_{kp})^* + (A_{kp})^*)\Phi^i\Phi^j\Phi^{\bar{p}} + h.c. -\\
&- \frac{g_s^{1/2}g_{YM}}{6}\epsilon_{ij3}S_{\bar{3}\bar{3}}\Phi^i\Phi^j\Phi^{\bar{3}} + h.c. - \frac{g_s^{1/2}g_{YM}}{9}G_{\bar{1}\bar{2}\bar{3}}\epsilon_{\bar{i}\bar{j}\bar{k}}\Phi^{\bar{i}}\Phi^{\bar{j}}\Phi^{\bar{k}} + h.c. -\\
&- g_{YM}^2[\Phi^i, \Phi^{(j}][\Phi^{\bar{j})}, \Phi^{\bar{i}}]\ \}
\end{aligned}$$

Here one defines $g_{YM}^2 \;=\; g_s(2\pi)^5(\alpha')^2\mathcal{V}^{-1}$, where $\mathcal{V}$ is the volume of the 4-torus

3.3.3 The T^4 case: fermionic sector

In the same way we one may in principle obtain the soft terms for the fermionic fields starting with the supersymmetrized DBI+CS actions in 8d. However the computation can be simplified carrying it out in 4d, and exploiting the symmetries of the system.

The four-dimensional fermionic action in the T^4 case can be obtained from the dimensional reduction of the supersymmetrized 10d action [30], up to quadratic order in the fermions, since we are interested in the fermion masses. Encoding the 4d fermions in a 10d Majorana-Weyl spinor Θ formed by the massless Ramond open string states, the $SO(3,1)\times SO(4)\times SO(2)$ invariant bilinears involving the 3-form flux are

$$\begin{aligned}
C\bar{\Theta}\Gamma^{mnp}\Theta[2ab(Re\ G_3)_{mnp} &- (a^2-b^2)(Im\ G_3)_{mnp}] + \qquad (3.20)\\
&+ C'\bar{\Theta}\Gamma_{(7)}\Gamma^{mnp}\Theta[(a^2-b^2)(Re\ G_3)_{mnp} - 2ab(Im\ G_3)_{mnp}]
\end{aligned}$$

where C and C' are two constants, and

$$\Gamma_{(7)} = \frac{-i}{8!}\epsilon_{ijklmnpq}\Gamma^i\Gamma^j\Gamma^k\Gamma^l\Gamma^m\Gamma^n\Gamma^p\Gamma^q \qquad (3.21)$$

with $i \ldots p$ running from 0 to 7. Also, a and b fix the embedding of the D7-brane supersymmetry in the 10d $\mathcal{N}=2$ IIB supersymmetry by means of $\theta_1 = a\sigma\Theta$, $\theta_2 = b\sigma\Theta$, where θ_1, θ_2, are the two Majorana-Weyl spacetime spinors of $\mathcal{N}=2$ supergravity and $a^2+b^2=1$. We work in the choice $(a,b)=(1,0)$.

The first term in (3.20) arises from contributions from the DBI piece of the eight dimensional supersymmetric action, whereas the second piece comes from CS contributions.

The fermionic masses in the four-dimensional action therefore are given in terms of the four adjoint fermions λ, Ψ^i, $i=1,2,3$, by

$$\begin{aligned}\mathcal{L}_F &= 6\sqrt{2}i(C'+C)[(G_{\bar{1}\bar{2}\bar{3}})^*\lambda\lambda + \frac{1}{2}(S_{\bar{3}\bar{3}})^*\Psi^3\Psi^3 + (A_{12})^*\Psi^3\lambda + \frac{1}{2}\sum_{ij=1,2} S_{ij}\Psi^i\Psi^j] + h.c. + \\ &\quad +6\sqrt{2}i(C'-C)[G_{123}\lambda\lambda + \frac{1}{2}S_{33}\Psi^3\Psi^3 + A_{\bar{1}\bar{2}}\Psi^3\lambda + \frac{1}{2}\sum_{\bar{i}\bar{j}=\bar{1},\bar{2}}(S_{\bar{i}\bar{j}})^*\Psi^i\Psi^j] + h.c. \quad (3.22)\end{aligned}$$

As a last step, the coefficients C and C' can be determined easily by supersymmetry arguments. An $S_{\bar{3}\bar{3}}$ background is ISD and primitive, and thus preserves unbroken $N=1$ supersymmetry. From (3.19) its scalar superpartner Φ^3 gets a mass term from such a background (which is one component in $(1,3')_-$), hence the fermion Ψ^3 should get an equal mass, which fixes $C+C' = -ig_s^{1/2}/72$. One may show from an analogous argument applied to $S_{\bar{1}\bar{2}}$, etc. that the second term in (3.22) is absent and hence $C=C'$. So finally, the four-dimensional fermionic masses are given by

$$\mathcal{L}_F = \frac{g_s^{1/2}}{6\sqrt{2}}Tr[(G_{\bar{1}\bar{2}\bar{3}})^*\lambda\lambda + \frac{1}{2}(S_{\bar{3}\bar{3}})^*\Psi^3\Psi^3 + (A_{12})^*\Psi^3\lambda + \frac{1}{2}\sum_{ij=1,2} S_{ij}\Psi^i\Psi^j] + h.c. \quad (3.23)$$

Again, note that only the fluxes transforming like $(3,1)_- + (1,3')_-$ appear, with $SU(2)\times SU(2)'\times U(1)$ R-symmetry relating independently λ with Ψ^3, and Ψ^1 with Ψ^2. In fact, the above fermion soft masses may be recast in $SO(4)\times SO(2)$ terms as follows

$$\mathcal{L}_F = \frac{g_s^{1/2}}{12}Tr[\,(\,\lambda\,,\ \Psi^3\,)\, i\sigma_y(G'^*\cdot\vec{\sigma})\begin{pmatrix}\lambda\\ \Psi^3\end{pmatrix} + (\,\Psi^1\,\Psi^2\,)\, i\sigma_y(\mathcal{G}^*\cdot\vec{\sigma})\begin{pmatrix}\Psi^1\\ \Psi^2\end{pmatrix}] + h.c. \quad (3.24)$$

3.3.4 Summary

Let us summarize the soft terms on the D7 worldvolume fields. In the notation defined in appendix I in ref [18], one gets

$$\begin{aligned} m^2_{1\bar{1}} &= m^2_{2\bar{2}} = 0 \ ; \quad B_{ij} \ = \ 0 \ \ , \ i,j\neq 3 \\ m^2_{3\bar{3}} &= \frac{g_s}{18}\left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}|S_{\bar{3}\bar{3}}|^2 + \frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2 + |A_{ij}|^2)\right)\end{aligned}$$

$$
\begin{aligned}
B_{33} &= \frac{g_s}{9}\left(\frac{1}{4}(S_{12})^{*2} + \frac{1}{4}(A_{12})^{*2} - \frac{1}{2}(G_{\bar{1}\bar{2}\bar{3}})^*(S_{\bar{3}\bar{3}})^* - \frac{1}{4}(S_{22})^*(S_{11})^*\right) \\
A^{ijk} &= -h^{ijk}\frac{g_s^{1/2}}{3\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^* \\
C^{ijk} &= -\frac{g_s^{1/2}}{6\sqrt{2}}\left[\sum_{l=1,2} h^{jkl}(S_{li}+A_{li}) - h^{jk3}(S_{\bar{3}\bar{3}})^*\right] \\
M^a &= \frac{g_s^{1/2}}{3\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^* \\
\mu_{33} &= -\frac{g_s^{1/2}}{6\sqrt{2}}(S_{\bar{3}\bar{3}})^* \\
\mu_{ij} &= -\frac{g_s^{1/2}}{6\sqrt{2}}S_{ij}\ ,\ \ i,j=1,2 \\
M_g^{3a} &= \frac{g_s^{1/2}}{6\sqrt{2}}(A_{12})^*
\end{aligned}
\tag{3.25}
$$

with

$$
h_{ijk} = 2\epsilon_{ijk}\sqrt{2}g_{YM,77} \tag{3.26}
$$

the Yukawa coupling. Again, recall that the above results provide the soft terms induced from pure ISD or pure IASD fluxes, since they do not take into account possible interference terms.

It is interesting at this point to compare the results obtained for soft terms in the case of D7-branes to those on D3-branes. Note the following points:

• Whereas only IASD fluxes G_{123}, S_{ij}, $A_{\bar{i}\bar{j}}$ contribute in the case of D3-branes, both ISD and IASD fluxes appear in the effective action for D7-branes. In particular note that the ISD flux $G_{\bar{1}\bar{2}\bar{3}}$ does appear in the case of D7-branes. This is a very important difference. Indeed, it is known that this kind of ISD background solves the Type IIB supergravity equations of motion in global compactifications. Thus, as we will emphasize in more detail later (see section 9), one can build orientifold models with D7-branes which are solutions of the classical equations of motion with zero vacuum energy but still contain explicit SUSY violating soft terms. This is not possible with only D3-branes.

• Only some of the components of the flux contribute to the soft terms, while for other components cancellation between the DBI and CS pieces of the action occur. This is consistent with the physical interpretation of soft terms as arising from interaction of lower-dimensional D-brane charges induced on the D7-branes with the background flux.

• Whereas in the case of D3-branes the fluxes do not contribute directly to scalar masses (they only do upon application of the equations of motion), in the case of D7-

branes we see that flux-dependent scalar masses and B-terms directly appear for the transverse scalars.

• Note also that in the D7-brane case (at least for the toroidal case which we are discussing now) the scalars $\Phi^{1,2}$ (Wilson line backgrounds) longitudinal to the D7-branes remain massless at this level. The underlying reason is as follows. The appearance of 8d mass terms for the internal components of the gauge fields is forbidden by 8d gauge invariance. Then, in the KK reduction on Σ_4, our assumption of translational invariance makes it impossible to obtain non-zero masses in the compactification. This is different for other 4-cycles, for instance K3, where no massless moduli associated to the D7-branes are present (even in the absence of fluxes). Moreover, we would like to emphasize that these Wilson line moduli may be absent in orbifold compactifications, due to the orbifold projection.

3.4 Solving the equations of motion

In the above results we have not yet fully imposed that the backgrounds solve the Type IIB supergravity equations of motion (although some relations, namely $Z_1 = Z_2$ and the dilaton being constant, have been taken into account). In the present section, we impose the remaining conditions, relating the warp factor and 5-form to the flux background, and show that those may be solved without affecting these results.

The relevant equations of motion for the dilaton τ, warping Z and 5-form F_5 are (see e.g.[18])

$$\begin{aligned} \frac{i}{2}\nabla^2\tau + \mathcal{O}((\nabla\tau)^2) &= \tfrac{1}{24}G_{mnp}G^{mnp} \\ 2\nabla^2 Z &= \tfrac{g_s}{12}G_{\bar{p}\bar{q}\bar{r}}G^*_{pqr} \\ dF_5 &= \tfrac{ig_s}{2}G_3\wedge G_3^* \end{aligned} \tag{3.27}$$

One can write

$$\frac{1}{24}G_{mnp}G^{mnp} = \frac{1}{2}\left(G_{123}G_{\bar{1}\bar{2}\bar{3}} + \frac{1}{4}S_{lk}S_{\overline{lk}} + \frac{1}{4}A_{lk}A_{\overline{lk}}\right)\ , \tag{3.28}$$

Therefore, the first equation is automatically satisfied since we have considered $\tau =$ const. and our flux background is purely ISD or purely IASD. Thus e.g. switching on $G_{\bar{1}\bar{2}\bar{3}}$ but not G_{123} and so on. This justifies a posteriori the type of fluxes that we describe in the first subsection of this chapter.

The other two equations yield

$$\begin{aligned} 2\nabla^2 Z &= \tfrac{g_s}{2}\left(|G_{123}|^2 + |G_{\bar{1}\bar{2}\bar{3}}|^2 + \tfrac{1}{4}\sum_{ij}(|S_{ij}|^2 + |S_{\bar{i}\bar{j}}|^2 + |A_{ij}|^2 + |A_{\bar{i}\bar{j}}|^2)\right) \\ 6\partial_{[\bar{3}}F_{1\bar{1}2\bar{2}3]} &= \tfrac{ig_s}{2}\left(|G_{123}|^2 - |G_{\bar{1}\bar{2}\bar{3}}|^2 + \tfrac{1}{4}\sum_{ij}(|S_{ij}|^2 - |S_{\bar{i}\bar{j}}|^2 - |A_{ij}|^2 + |A_{\bar{i}\bar{j}}|^2)\right) \end{aligned} \tag{3.29}$$

These are easily solved within the ansatz that the warp factor A and 5-form are independent of the coordinates along the 4-cycle, and depend only on z_3, $\overline{z}_3$. This leads to

$$\begin{aligned} 4K_{3\bar{3}} &= \tfrac{g_s}{2}\left(|G_{123}|^2 + |G_{\bar{1}\bar{2}\bar{3}}|^2 + \tfrac{1}{4}\textstyle\sum_{ij}(|S_{ij}|^2 + |S_{\overline{ij}}|^2 + |A_{ij}|^2 + |A_{\overline{ij}}|^2)\right) \\ -2\chi_{3\bar{3}} &= \tfrac{g_s}{2}\left(|G_{123}|^2 - |G_{\bar{1}\bar{2}\bar{3}}|^2 + \tfrac{1}{4}\textstyle\sum_{ij}(|S_{ij}|^2 - |S_{\overline{ij}}|^2 - |A_{ij}|^2 + |A_{\overline{ij}}|^2)\right) \end{aligned} \quad (3.30)$$

In this way all equations of motion are solved. Note that this is consistent with the type of closed string background (3.10), (3.11) discussed at the beginning of this chapter. In particular, in the present simple solution the 3-form fluxes give rise to a warp factor and 5-form depending only in the dimensions transverse to the D7-branes. Note on the other hand that the warp factor and 5-form do not appear in the expressions for the soft terms found above, and hence solving the equations of motion does not give extra constraints in the form of the soft term Lagrangian. This is related to the fact that D7-branes are not charged under C_4, and that the gravitational effect of pure ISD or pure IASD fluxes (which have $Z_1 = Z_2$) on D7-branes cancel.

These are not the most general class of solutions of the equations of motion, and choosing Z and F_5 not depending on z^1, z^2 is rather a simplifying assumption in order to facilitate the computation of the soft terms on the D7-brane worldvolume. General features beyond this simplified situation are described in section 8.

Note that in the simpler computation of soft terms on D3-branes in section 2 [18], the problem can be solved for arbitrary coordinate dependence for the warp factor and 5-form (or even the dilaton). If one however replaces the above more restricted background, constant over the four coordinates, the equations of motion determine the masses for the scalars (and not only its trace). In particular from eq.(2.7) we obtain that the only non-vanishing scalar mass is that of the scalar $\Phi^3_{(33)}$ parametrizing the D3-brane position in the third complex dimension

$$m^2_{(33)} \;=\; \frac{g_s}{2}\left[\,|G_{123}|^2 \,+\, \frac{1}{4}\sum_{ij}\left(\,|S_{ij}|^2 \,+\, |A_{\overline{ij}}|^2\,\right)\right] \qquad (3.31)$$

The masslessness of Φ^1, Φ^2 simply means that there is no preferred position for the D3-brane in this restricted situation of translational invariance along T^4

3.5 T^4 with variable warp factor

We would like to conclude with some remarks on the results in the slightly more general (and more realistic) situation where the background (namely the warp factor and 5-form) varies along the 4-cycle on which the D7-branes wrap. This may be the case

even for T^4, as long as the local background satisfies the corresponding periodicity conditions. In fact, one expects this to be the situation in any compact model, since different ingredients, away from the D7-branes (like distant O3-planes, D3-branes or other 7-branes) in general distort the warp factor at the location of the D7-branes, so that it it not constant over T^4.

Clearly, the computation of the 8d action, including the effects of the backgrounds, is very similar to the above one, and simply differs in the appearance of new derivative couplings, etc. The KK compactification proceeds as above, the only difference arising from the fact that the backgrounds cannot be pulled out of the integrals over the 4-cycle. This implies that the resulting 4d soft terms have exactly the same structure as the above computed ones, but their coefficients are integrals of the backgrounds over T^4, suitably convoluted with the internal wavefunction of the 4d zero modes. Since for T^4 the latter are constant, the coefficients of the soft terms are simply the average values of the corresponding background over the internal 4-cycle. Hence, all the above expressions remain valid, with the understanding that the constant flux coefficients now correspond to their average values over the internal 4-cycle.

One may think that the introduction of varying backgrounds may lead to qualitative changes in the special sector of the Wilson line scalars. Indeed our arguments above for their masslessness relied on 8d gauge invariance, plus the trivial KK reduction over T^4. It could be expected that a varying background would introduce new derivative couplings for the 8d gauge bosons, leading to non-trivial 4d mass terms for these Wilson line degrees of freedom. However this expectation is too naive, and not correct: the number of massless 4d scalars from Wilson line degrees of freedom is determined topologically, as the first Betti number of the internal 4-cycle, i.e. as the number of zero modes of the laplacian for 1-forms. The appearance of additional derivative couplings corresponds to a change in this internal laplacian, but which cannot change its number of zero modes. This implies that the internal wavefunction of the 4d Wilson line scalars rearranges in such a way that the latter remain massless [12]. Hence, this relevant feature of the toroidal case is not changed by considering more general backgrounds.

[12]Note that this implies that the coefficient of the scalar trilinear soft terms is not exactly the average value of the flux, but rather has a non-trivial convolution with the corrected internal profiles for Wilson line scalars.

4 Soft terms at $D3 - D7$-brane intersections

We will now consider the combined system of D3-branes and D7-branes, at equal position in the common transverse third complex plane, $\langle \Phi_{33}^3 \rangle = \langle \Phi_{77}^3 \rangle$. The effect of fluxes in the new sector of 37 fields is essentially different from the previous cases, and notoriously more difficult, since there is no analog of the DBI+CS action that describes the coupling of these fields to a general supergravity background. Nevertheless, the system has a large enough symmetry to constrain the possible lower-dimension terms, so that with additional partial information we are able to fully determine them [13]. This is our purpose in the present section.

Since 37 states are localized at points in the internal manifold, one may carry out the analysis with a local configuration, without full information of the 4-cycle spanned by the D7-branes. Nevertheless, for consistency with our previous discussion we consider flux backgrounds of the kind there considered.

The local configuration is described by a set of D3-branes, and a set of D7-branes spanning the coordinates z_1, z_2, in a (suitably backreacted) flat space, described by the ansatz (3.10). The local geometric symmetry is $SU(2) \times SU(2)' \times U(1)$, and must be respected by the soft terms. In the absence of fluxes, the system preserves 8 supercharges, namely $N = 2$ supersymmetry in 4d. The 37 fields localized at the D3-brane position are four-dimensional, and fill out a hypermultiplet of this supersymmetry. Described in terms of $N = 1$ supersymmetry [14] , we have two $N = 1$ chiral multiplets (Φ_{37}, Ψ_{37}) and (Φ_{73}, Ψ_{73}), in conjugate representations of the gauge group. Under the $SU(2) \times SU(2)' \times U(1)$ symmetry, the complex scalars Φ_{37} and Φ_{73}^* transform as $(1, 2')_0$, whereas the fermions transform as $(1, 1)_{\pm\frac{1}{2}}$.

The system has SUSY superpotential couplings of the form

$$W_{37} \;=\; \Phi_{77}^3 \Phi_{73} \Phi_{37} \;-\; \Phi_{33}^3 \Phi_{37} \Phi_{73} \tag{4.1}$$

corresponding to the fact that separating the D3- and D7-brane locations in the third complex plane gives mass to the hypermultiplet at the intersection.

The possible soft terms involving the fields at the intersections are very much restricted by the local geometric symmetries of the system. Since the scalars belong to doublets of $SU(2)'$, bilinears for these scalars may only come from 3-form backgrounds

[13] It may be possible to exploit the alternative approach in [31] to achieve these results.

[14] For $\overline{D3}$/D7-brane systems, to be discussed in section 6, the system preserves a different $N = 1$ supersymmetry. The discussion is similar, but with an exchange of the roles of the two $SU(2)$ symmetries.

transforming like $(1,3')_-$ or $(1,3')_+$. This means backgrounds of type $(G_{\bar{1}\bar{2}\bar{3}}, A_{12}, S_{\bar{3}\bar{3}})$ or else $(G_{123}, A_{\bar{1}\bar{2}}, S_{33})$. In fact, we will describe below which explicit couplings of these form are indeed present, using powerful symmetry arguments. However it is interesting to motivate their presence from an independent point of view, as follows.

Although we lack the general action describing the coupling of 37 fields to a general supergravity background, there is some partial knowledge about some of these couplings. Namely, the presence of a NSNS 2-form field along the D7-brane modifies the boundary conditions of open strings ending on it, and hence induces a change in the mass of scalars from 37 strings. Such mass terms can be explicitly obtained by computing the open string spectrum in the presence of this background, using standard world-sheet techniques. Considering a general B-field background along the D7-brane, one may use the $SO(4)$ rotation group to bring it to a block diagonal form, where only the components $B_{1\bar{1}}$, $B_{2\bar{2}}$ are turned on. The contribution of this background to masses of 37 states is described by the mass term in the 4d action of the system

$$\frac{i}{2\pi\alpha'}(\, B_{1\bar{1}} + B_{2\bar{2}}\,)\,(\Phi_{73}\,\Phi_{73}^* + \Phi_{37}\Phi_{37}^*) \,+\, \text{h.c.} \tag{4.2}$$

Note that there are no terms involving the 37 fermion fields. As mentioned above, this describes the leading term coupling the 37 fields to a general NSNS 2-form background, and plays the role of the DBI piece of the action for 33 and 77 fields.

According to the general philosophy, we may now replace in the above general term the B-field that corresponds to a specific 3-form flux background. Namely, at leading order in the coordinates, we have

$$B_{mn} \;=\; -\frac{g_s}{6i}(G_{mnp} \;-\; G^*_{mnp})x^p \tag{4.3}$$

In particular using (3.6), the combination $(B_{1\bar{1}} + B_{2\bar{2}})$ is related to the A_{12} 3-form flux component, since

$$B_{1\bar{1}} \;+\; B_{2\bar{2}} \;=\; -\frac{g_s}{6i}\left(A_{12}\bar{z}^3 \;-\; A_{\bar{1}\bar{2}}z^3 \;+\; (A_{12})^* z^3 \;-\; (A_{\bar{1}\bar{2}})^*\bar{z}^3\right) \tag{4.4}$$

Replacing this in (4.2), and trading z^3 for the D7-brane world-volume scalar in the third complex plane, we obtain the trilinear soft term

$$\frac{g_s}{6}[(A_{12})^* - A_{\bar{1}\bar{2}}]\Phi_{77}^3\Phi_{73}\Phi_{73}^* \;+\; h.c \tag{4.5}$$

And as discussed above, there are no fermion mass terms. Notice that, as announced, the coupling involves a flux component in the $(1,3')_-$ representation of $SU(2)\times SU(2)'\times U(1)$. Now, although we computed the above term for a particular flux component,

we may covariantize the expression with respect to $SO(4)$ to reach the general result of soft terms induced by a general NSNS 3-form flux.

Note however that this analysis is suggestive but not conclusive, since we have no information about the coupling of 37 fields to RR fields (say, the analog of the CS couplings for 33 and 77 sectors). And these are crucial, since they may (and do, see later) lead to cancellations with terms from coupling to NSNS backgrounds. Hence we conclude that the above approach is not sufficient to obtain the complete soft terms for general NSNS/RR flux backgrounds.

Fortunately, the complete answer can be instead found by fully exploiting the symmetries of the system, namely supersymmetry of some particular backgrounds, and the full $SU(2) \times SU(2)' \times U(1)$ geometric symmetry. This can be exploited to obtain the full soft term lagrangian for 37 fields for a general flux background, as follows.

Consider the D3/D7-brane system in the particular background 3-form flux $S_{\bar{3}\bar{3}}$, which transforms in the $(1, 3')_-$ representation. This is a primitive (2,1)-form flux, hence the combined system preserves 4d $N = 1$ supersymmetry. Indeed, in the 77 sector we have seen from (3.25) that such a background gives rise to a superpotential mass term for the chiral multiplet containing Φ^3_{77}, Ψ^3_{77},

$$W^{(77)}_\mu \ = \ -\frac{1}{2}\mu_{(77)}\Phi^3_{77}\Phi^3_{77} \ = \ \frac{g_s^{1/2}}{12\sqrt{2}}(S_{\bar{3}\bar{3}})^*\Phi^3_{77}\Phi^3_{77} \tag{4.6}$$

On the other hand the Φ^3_{77} scalar has a superpotential coupling as in (4.1). Thus the relevant piece of the scalar potential is

$$V \ = \ |\mu_{(77)}\Phi^3_{77} \ - \ \Phi_{73}\Phi_{37}|^2, \tag{4.7}$$

whose crossed term yields a trilinear coupling

$$\frac{g_s^{1/2}}{6\sqrt{2}}(S_{\bar{3}\bar{3}})^*\Phi^3_{77}\Phi^*_{73}\Phi^*_{37} \ + \ h.c. \tag{4.8}$$

This should be interpreted as the effect of the 3-form flux background $S_{\bar{3}\bar{3}}$ on 37 fields. Notice that it has a structure reminiscent of the trilinear couplings determined above, but the present expression, determined purely from supersymmetry considerations, automatically contains contributions from both the coupling to the NSNS and RR backgrounds (and hence accounts for possible cancellations between them).

Now we may use the full $SU(2) \times SU(2)' \times U(1)$ symmetry to covariantize the above expression, using the fact that $S_{\bar{3}\bar{3}}$ is a component of a triplet $(1, 3')_-$ of fluxes, which also includes $G_{\bar{1}\bar{2}\bar{3}}$ and A_{12}. The complete expression for the soft terms induced by a

general flux in the $(1,3')_-$, is

$$\frac{g_s^{1/2}}{6\sqrt{2}}[(S_{\bar{3}\bar{3}})^*\Phi_{77}^3\Phi_{73}^*\Phi_{37}^* \;+\; 2(G_{\bar{1}\bar{2}\bar{3}})^*\Phi_{77}^3\Phi_{73}\Phi_{37} \;+ \\ +\; (A_{12})^*\Phi_{77}^3\Phi_{37}\Phi_{37}^* \;+\; (A_{12})^*\Phi_{77}^3\Phi_{73}\Phi_{73}^* \;+\; h.c.] \tag{4.9}$$

Note how indeed the first trilinear scalar coupling advanced in (4.5) reappears here from a completely different argumentation, while the second is not present (hence due to a cancellation between the NSNS and RR flux contributions, automatically taken care of by our analysis). The soft terms written in $SO(4)\times SO(2)$ notation read

$$\frac{g_s^{1/2}}{6}(\Phi_{37}^* \quad \Phi_{73})(G'^*\cdot\vec{\sigma})\begin{pmatrix}\Phi_{37}\\ \Phi_{73}^*\end{pmatrix}\Phi_{77}^3 \tag{4.10}$$

We may use a similar strategy to compute the soft terms induced from flux backgrounds in the $(1,3')_+$ representation. Let us start now from a pure S_{33}, which is primitive (1,2) and hence IASD. This flux does not preserve SUSY, but it was found in [18] (see eq.(3.14) in that reference) that such type of background gives rise to a supersymmetric mass term for the chiral multiplet containing the D3-brane fields Φ_{33}^3, Ψ_{33}^3, (see also [26])

$$W_\mu^{(33)} \;=\; -\frac{1}{2}\mu_{(33)}\Phi_{33}^3\Phi_{33}^3 \;=\; \frac{g_s^{1/2}}{4\sqrt{2}}S_{33}\Phi_{33}^3\Phi_{33}^3 \tag{4.11}$$

On the other hand the Φ_{33}^3 scalar has a superpotential coupling as in eq.(4.1). Thus in the scalar potential there is a term

$$V \;=\; |\mu_{33}\Phi_{33}^3 \;+\; \Phi_{37}\Phi_{73}|^2 \tag{4.12}$$

whose crossed term yields a trilinear coupling

$$-\frac{g_s^{1/2}}{2\sqrt{2}}S_{33}\Phi_{33}^3\Phi_{37}^*\Phi_{73}^* \;+\; h.c. \tag{4.13}$$

Again, we may now use the $SU(2)\times SU(2)'\times U(1)$ symmetry to covariantize the above expression, and obtain the general soft terms for 37 fields induced by a general flux in the $(1,3')_+$ representation. The complete trilinear scalar couplings reads

$$\frac{g_s^{1/2}}{2}(\Phi_{37} \quad \Phi_{73}^*)(\mathcal{G}'\cdot\vec{\sigma})\begin{pmatrix}\Phi_{37}^*\\ \Phi_{73}\end{pmatrix}\Phi_{33}^3 = \tag{4.14}$$

$$= \;-\frac{g_s^{1/2}}{2\sqrt{2}}\left[S_{33}\Phi_{33}^3\Phi_{37}^*\Phi_{73}^* \;+\; 2G_{123}\Phi_{33}^3\Phi_{37}\Phi_{73} \;+\; A_{\bar{1}\bar{2}}\Phi_{33}^3\Phi_{73}\Phi_{73}^* \;+\; A_{\bar{1}\bar{2}}\Phi_{33}^3\Phi_{37}\Phi_{37}^* \;+\; h.c.\right]$$

The expressions (4.10) and (4.15) provide the complete set of soft terms induced by a general 3-form flux configuration (since fluxes in the $(3,1)$ cannot couple to 37 scalars, as discussed above). Note also that, as a general property, trilinear terms involving the D7-brane coordinates Φ_{77}^3 are sourced by ISD fluxes, whereas those involving the D3-brane coordinates Φ_{33}^3 are sourced by IASD fluxes. This is in full agreement with the results from the low-energy 4d effective supergravity Lagrangian, in section 7.

Finally, notice that no scalar or fermion masses are generated. This is somehow expected from our previous experience, as follows. Fields in 37 sectors can be regarded as moduli of an instanton bundle, parametrizing the possibility of transforming the D3-branes into finite-size instantons on the D7-brane world-volume. Hence, their dynamics is reminiscent of other bundle moduli, like Wilson lines, and do not get masses, for reasons already explained in section 3.3.4.

5 Examples of fluxes

Let us now discuss some particular examples of flux choices.

5.1 ISD fluxes

5.1.1 (0,3) backgrounds

Let us consider first the case in which only the $G_{\bar{1}\bar{2}\bar{3}}$ background is switched on. This is a $(0,3)$ ISD form which is of particular relevance since it was shown in [6] that such a background solves the equations of motion with vanishing cosmological constant [15]. From the results in [18] reviewed at the beginning of section 2, no soft terms appear for the D3-brane worldvolume fields. On the other hand from the results in previous sections, fields from both 77 and 37 sectors acquire soft terms, as follows

$$m^2_{\Phi^3_{77}} \;=\; \frac{g_s}{18}\,|G_{\bar{1}\bar{2}\bar{3}}|^2 \;;\; M^{(77)} \;=\; \frac{g_s^{1/2}}{3\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^* \;;\; A^{ijk\ (77)} \;=\; -h^{ijk}\,\frac{g_s^{1/2}}{3\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^* \quad (5.1)$$

where $A^{ijk\ (77)}$ is the coefficient of the trilinear scalar coupling proportional to $\Phi_{77}^3\Phi_{77}^2\Phi_{77}^1$, $h_{ijk} = 2\sqrt{2}g_{YM}\epsilon_{ijk}$ is the corresponding Yukawa coupling, and M^{77} are D7-brane gaugino masses. The rest of the 77 soft terms vanishes. Note the relations

$$A^{ijk\ (77)} \;=\; -h^{ijk}M^{(77)} \;;\; m^2_{\Phi^3_{77}} \;=\; |M^{(77)}|^2 \quad (5.2)$$

[15]Note that the IASD backgrounds considered above do also solve the supergravity equations of motion in the local context, but cannot be extended to compact spaces keeping vanishing potential energy.

In addition there is a trilinear scalar coupling involving the 37 scalars and the D7-brane scalar Φ_{77}^3

$$A_{\Phi_{77}^3(73)(37)} \; = \; -\frac{g_s^{1/2}}{3\sqrt{2}}(G_{\bar{1}\bar{2}\bar{3}})^* \tag{5.3}$$

Note that these type of soft terms are somewhat analogous to the 'dilaton-dominated' type of terms which appear for $D3$-fields in an IASD (3,0) background [18]. However an important difference is that now we have an ISD (0,3) background, which solves the equations of motion with vanishing cosmological constant, even in compact models.

5.1.2 ISD $S_{\bar{3}\bar{3}}$ flux

This is an ISD primitive $(2,1)$ form and hence we know from general arguments that they preserve one unbroken SUSY. Thus they may give rise at most to additional superpotential interactions. From the analysis in [18], such type of backgrounds give no soft terms for 33 fields, which are only sensitive to IASD fluxes. On the other hand, for D7-branes, a $S_{\bar{3}\bar{3}}$ flux gives rise to a superpotential term, see eqs.(3.25)

$$W_\mu^{(77)} \; = \; -\frac{1}{2}\mu_{(77)}\Phi_{77}^3\Phi_{77}^3 \; = \; \frac{g_s^{1/2}}{12\sqrt{2}}(S_{\bar{3}\bar{3}})^*\Phi_{77}^3\Phi_{77}^3 \tag{5.4}$$

Note that if both $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$ fluxes are present there is an additional soft bilinear B-term (see eq.(3.25)) for the scalars Φ_{77}^3 given by

$$B_{33} \; = \; -\frac{g_s}{18}\,(G_{\bar{1}\bar{2}\bar{3}})^*(S_{\bar{3}\bar{3}})^* \; = \; 2M\mu_{(77)} \tag{5.5}$$

It is interesting to point out that the full set of bosonic ISD soft terms may be combined with the SUSY F-term scalar potential to obtain a perfect square positive definite scalar potential as follows

$$V_{ISD} \; = \; |\,-M_{77}^*{\Phi_{77}^3}^* - \mu_{(77)}\Phi_{77}^3 + \Phi_{77}^1\Phi_{77}^2 + \Phi_{73}\Phi_{37}\,|^2 \; = \; |\,-M_{77}^*{\Phi_{77}^3}^* + \partial_{\phi^3}W\,|^2 \tag{5.6}$$

with W the full superpotential. Indeed it is easy to check that all bosonic soft terms described above from ISD backgrounds are reproduced in this way. The origin of this positive definitive term may be traced back to a piece proportional to $|B_2 \,-\, F_2|^2$ in the DBI action in eq.(3.16). Indeed, for these backgrounds $B_2 \simeq G_{\bar{1}\bar{2}\bar{3}}\bar{z}^3 + (S_{\bar{3}\bar{3}})^*z^3$. Upon replacing $z^3 \to \phi_{77}^3$, and taking into account that upon dimensional reduction F_2 gives rise to the SUSY F-term (contributing the familiar commutator squared to the scalar potential), the above structure is obtained. The appearance of this positive definite scalar potential has also a clear interpretation in terms of the no-scale structure of

ISD backgrounds in the effective supergravity action, as we show in section 7, see eq. (7.14).

The above expression of the scalar potential for D7-brane matter fields as a positive definite quantity is also closely related to the description of the above configurations in F-theory. Indeed, a configuration of n D7-branes on a 4-cycle Σ_4 in a threefold B_3, can be described as F-theory on a 4-fold, elliptically fibered over B_3, and with an I_{n+1} Kodaira degeneration of the elliptic fiber over the divisor Σ_4. Equivalently, as M-theory on the same fourfold, in the limit of shrinking the size of the elliptic fiber (and blowing down the exceptional divisors associated to the singularity from the degeneration). The D7-brane geometric moduli, i.e. moduli of the 4-cycle Σ_4 correspond to complex structure deformations of the F/M-theory fourfold, describing the geometry of the degenerate fiber locus. Turning on a general ISD complex 3-form flux G_3 on the IIB picture corresponds to turning on a general real self-dual 4-form flux G_4 in F/M-theory, given by

$$G_4 \,=\, \frac{G_3\, d\bar{w}}{\bar{\tau}-\tau} \,+\, \text{h.c.} \tag{5.7}$$

where $dw = dx + \tau dy$ and τ is the IIB complex coupling. Hence $(2,1)$ and $(0,3)$ G_3 flux components correspond to $(2,2)$ and $(0,4)$ (plus h.c.) G_4 flux components. In the F/M-theory picture, we have just a 4-form flux in a CY fourfold. This system has been studied in [2] and is described by a superpotential [3]

$$W \,=\, \int_{CY_4} G_4 \wedge \Omega_4 \tag{5.8}$$

where $\Omega_4 = \Omega_3 dw$ is the holomorphic 4-form in the fourfold. This superpotential leads to a positive definite potential for the scalars, which in particular include the D7-brane geometric moduli. Hence this confirms the above result for the scalar potential for D7-brane moduli in ISD 3-form fluxes. The connection has been very explicitly shown in [21]. It is very satisfactory to recover the F/M-theory picture from considerations of the D7-brane action.

5.2 IASD backgrounds

5.2.1 $(3,0)$ fluxes

For this kind of backgrounds the effect of the flux on D3- and D7-fields is reversed. Indeed, we saw that a background G_{123} does not give rise to any soft terms on the D7-brane worldvolume fields. On the other hand they give rise [18] to soft terms for

fields in the 33 sector

$$m^2_{\Phi^3_{33}} \;=\; \frac{g_s}{2}\,|G_{123}|^2 \;;\; M^{(33)} \;=\; \frac{g_s^{1/2}}{\sqrt{2}}\,G_{123} \;;\; A^{ijk\ (33)} \;=\; -h^{ijk}\,\frac{g_s^{1/2}}{\sqrt{2}}\,G_{123} \qquad (5.9)$$

where $M^{(33)}$ are the D3-brane gaugino masses. In addition there is a trilinear coupling involving the D3-brane scalar Φ^3_{33} and the 37 scalars

$$A_{\Phi^3_{33}(73)(37)} \;=\; -\,\frac{g_s^{1/2}}{\sqrt{2}}G_{123} \qquad (5.10)$$

The rest of soft terms vanish. In analogy with the previous case, there are no scalar masses for the D3-brane fields Φ^1_{33}, Φ^2_{33} due to our simplifying assumptions on the coordinate dependence of the background. Again, this would change for more general backgrounds.

The interesting symmetry of soft terms under the exchange $D3 \leftrightarrow D7$, $G_{\bar{1}\bar{2}\bar{3}} \leftrightarrow G_{123}$ may be understood from the effective $D = 4$ supergravity effective Lagrangian approach, as we describe in section 7.

5.2.2 S_{33} flux

We saw in previous section that such a background does not contribute to the soft term Lagrangian for D7-brane fields. On the other hand, as we mentioned above, this kind of flux gives rise to a SUSY superpotential mass term for D3-brane fields

$$W_\mu^{(33)} \;=\; -\frac{1}{2}\mu_{(33)}\Phi^3_{33}\Phi^3_{33} \;=\; \frac{g_s^{1/2}}{4\sqrt{2}}S_{33}\Phi^3_{33}\Phi^3_{33} \qquad (5.11)$$

Again this is the only term from such a flux. This is in agreement with the symmetry under exchanges $D3 \leftrightarrow D7$ and ISD $\leftrightarrow$ IASD, mentioned above, see section 7.

6 Fluxes and $\overline{D3} - D7$ brane intersections

Some interesting Type IIB compactifications make also use of antibranes. Thus it is interesting to consider replacing $D3$-branes by $\overline{D3}$'s embedded inside D7-branes. Now, as emphasized in [18] the effect of fluxes on $\overline{D3}$'s is obtained from that with $D3$-branes by replacing $ISD \leftrightarrow IASD$ fluxes. The effect of the fluxes on the fields at the $\overline{D3} - D7$ intersections may be directly derived from those in $D3 - D7$ by symmetry considerations. Indeed, the system $\overline{D3} - D7$ preserves a different $N = 2$ supersymmetry compared to that of D3-D7 system. The (Ψ_1, Ψ_2) fermions on $D7$-branes are now inside a $N = 2$ vector multiplet whereas those we labeled as (Ψ_3, λ)

belong to a hypermultiplet. Thus considering the $SO(4) = SU(2)\times SU(2)'$ symmetries the vector fermions belong to a $(2,1)$ whereas those in hypermultiplets belong to a $(1,2')$. The scalars at the $\overline{D3}-D7$ intersections $\Phi_{3\bar{7}}$ and $\Phi^*_{7\bar{3}}$ form a an $SU(2)$ doublet $(2,1)$ rather than a $(1,2')$. As a consequence only $(3,1)_-$ and $(3,1)_+$ can couple to these scalars. The trilinear couplings for $(\bar{3}7)$ scalars are obtained from those for (37) scalars by replacing the fluxes as follows

$$
\begin{aligned}
(1,3')_- : \quad & (A_{12}, S_{\bar{3}\bar{3}}, 2G_{\bar{1}\bar{2}\bar{3}}) \;\longrightarrow\; (3,1)_- : \quad (-S_{12}, S_{11}, S_{22}) \\
(1,3')_+ : \quad & (A_{\bar{1}\bar{2}}, S_{33}, 2G_{123}) \;\longrightarrow\; (3,1)_+ : \quad (-S_{\bar{1}\bar{2}}, S_{\bar{1}\bar{1}}, S_{\bar{2}\bar{2}})
\end{aligned}
\tag{6.1}
$$

We thus get trilinear couplings to IASD fluxes given by

$$
\begin{aligned}
\frac{g_s^{1/2}}{6\sqrt{2}}[(S_{11})^*\Phi^3_{77}\Phi^*_{7\bar{3}}\Phi^*_{\bar{3}7} \quad & + \quad (S_{22})^*\Phi^3_{77}\Phi_{7\bar{3}}\Phi_{\bar{3}7} \;- \\
& - \quad (S_{12})^*\Phi^3_{77}\Phi_{\bar{3}7}\Phi^*_{\bar{3}7} \;-\; (S_{12})^*\Phi^3_{77}\Phi_{7\bar{3}}\Phi^*_{7\bar{3}} \;+\; h.c.]
\end{aligned}
\tag{6.2}
$$

whereas ISD fluxes yield

$$
-\frac{g_s^{1/2}}{2\sqrt{2}}\left[S_{\bar{1}\bar{1}}\Phi^3_{\bar{3}\bar{3}}\Phi^*_{\bar{3}7}\Phi^*_{7\bar{3}} \;+\; S_{\bar{2}\bar{2}}\Phi^3_{\bar{3}\bar{3}}\Phi_{\bar{3}7}\Phi_{7\bar{3}} \;-\; S_{\bar{1}\bar{2}}\Phi^3_{\bar{3}\bar{3}}\Phi_{7\bar{3}}\Phi^*_{7\bar{3}} \;-\; S_{\bar{1}\bar{2}}\Phi^3_{\bar{3}\bar{3}}\Phi_{\bar{3}7}\Phi^*_{\bar{3}7} \;+\; h.c.\right]
\tag{6.3}
$$

Note that e.g. in the interesting case of a ISD $(0,3)$ flux background $G_{\bar{1}\bar{2}\bar{3}}$ no soft terms appear for fields at the intersections.

7 Comparison with effective supergravity Lagrangian approach

7.1 The D3/D7-brane system

Soft terms may also computed in the formalism in [32, 33, 34], from vevs for auxiliary fields for 4d moduli in the 4d effective supergravity action. As described in [18, 19], 3-form fluxes breaking SUSY correspond to non-vanishing auxiliary fields for the dilaton and/or moduli fields of $N = 1$ preserving CY Type IIB orientifold compactifications. Hence, we may use the formalism of the 4d effective action to compute soft terms on gauge sectors of these compactifications, and compare the results with our above local analysis.

Such compactifications always include a massless complex dilaton chiral field

$$
S \;=\; -i\tau \quad ; \quad \tau \;=\; C \;+\; i/g_s \;, \tag{7.1}
$$

whose imaginary part is related to the dilaton and C is the type IIB axion. In addition there will be a number of complex structure scalars M_a and Kähler moduli T_α. Consider for simplicity the dependence on a single overall Kähler modulus T, whose real part gives the overall radius of the compactification. In the large volume limit the Kähler potential has the well known form

$$\frac{K}{M_p^2} \;=\; -\log(S+S^*) \;-\; 3\log(T+T^*) \tag{7.2}$$

where M_p^2 is the 4d Planck mass squared. We consider compactifications with a non-trivial background for the NSNS and RR field strength 3-forms $H_{(3)}$,$F_{(3)}$. In this situation an S-dependent effective superpotential is given by [3, 5]

$$W \;=\; \kappa_{10}^{-2}\int G_{(3)}\wedge\Omega \;\;,\;\; G_{(3)} \;=\; F_{(3)} - iSH_{(3)}\;, \tag{7.3}$$

where $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7\alpha'^4$ is the $D=10$ gravitational constant and Ω the Calabi-Yau holomorphic 3-form. One can show that standard supergravity formulae then yield for the auxiliary fields of T and S

$$\begin{aligned} F^S &= \tfrac{1}{M_p^2}\,(S+S^*)^{1/2}\,(T+T^*)^{-3/2}\,(\kappa_{10}^{-2})\textstyle\int {G_{(3)}}^*\wedge\Omega \\ F^T &= -\tfrac{1}{M_p^2}\,(S+S^*)^{-1/2}\,(T+T^*)^{-1/2}\,(\kappa_{10}^{-2})\textstyle\int G_{(3)}\wedge\Omega \end{aligned} \tag{7.4}$$

Thus, e.g., a flux background of $(3,0)$ type would then correspond to a non-vanishing auxiliary field for the complex dilaton S, whereas a $(0,3)$ background would correspond to the overall Kähler field T. The gravitino mass is given by

$$m_{3/2}^2 \;=\; \frac{e^{K/M_p^2}}{M_p^4}\,|W|^2 \;=\; \frac{1}{M_p^4}\,(S+S^*)^{-1}\,(T+T^*)^{-3}\,(\kappa_{10}^{-4})\,|\int G_{(3)}\wedge\Omega\,|^2 \tag{7.5}$$

Note that for a $(0,3)$ background supersymmetry is broken with a vanishing c.c. (no-scale structure) whereas for a $(3,0)$ background there is a positive c.c. given by

$$V_0 \;=\; \frac{\kappa_{10}^{-4}|\int G_{(3)}^*\wedge\Omega|^2}{M_p^2(S+S^*)(T+T^*)^3}. \tag{7.6}$$

with zero gravitino mass.

Consider now such Type IIB backgrounds in the presence of $D7$- and $D3$-branes. Although many of the points we will discuss are more general, let us consider first the case of a toroidal compactification on a factorized $T^6 = T^2\times T^2\times T^2$. Let us concentrate on the dilaton and the three T_i, $i=1,2,3$ Kähler moduli which control the size of the tori. Consider now a system of D3- and/or D7-branes (wrapping the

first two tori). This is the simplest configuration that we studied in the first sections of this paper. Now it was found in ref.[23] (see also [24]) that the gauge kinetic functions for the $D7(D3)$ Lagrangian $f_7(f_3)$ as well as the Kähler potential depend on S, T_i as follows:

$$
\begin{aligned}
f_3 &= S \quad , \quad f_7 = T_3 \, , \\
K &= -\log(S+S^* - |\Phi^3_{77}|^2) - \log(T_3 + T_3^* - |\Phi^3_{33}|^2) \\
&- \log(T_2 + T_2^* - |\Phi^2_{33}|^2 - |\Phi^1_{77}|^2) - \log(T_1 + T_1^* - |\Phi^1_{33}|^2 - |\Phi^2_{77}|^2) \\
&+ \frac{|\Phi_{37}|^2 + |\Phi_{73}|^2}{(T_1+T_1^*)^{1/2}(T_2+T_2^*)^{1/2}} \, ,
\end{aligned}
\tag{7.7}
$$

Focusing only on the dependence on the dilaton S and overall Kähler modulus T this yields for large T

$$
\begin{aligned}
f_3 &= S \quad , \quad f_7 = T \, , \\
K &= -\log(S+S^*) - 3\log(T+T^*) \qquad (7.8) \\
&+ \frac{|\Phi^3_{77}|^2}{(S+S^*)} + \frac{1}{(T+T^*)}\left[\left(\sum_{a=1}^{3} |\Phi^a_{33}|^2\right) + \left(\sum_{b=1}^{2} |\Phi^b_{77}|^2\right) + \left(|\Phi_{37}|^2 + |\Phi_{73}|^2\right) \right]
\end{aligned}
$$

From this and the results for the auxiliary fields eq.(7.4) one can compute the SUSY-breaking soft terms for the case of ISD $(0,3)$ and IASD $(3,0)$ fluxes, which correspond to $(F_S = 0, F_T \neq 0)$ and $(F_S \neq 0, F_T = 0)$ respectively by using standard supergravity formulae. In fact the results may be directly read out from section (7.2) of reference [23] [16]. Although the results for the soft terms in terms of the auxiliary field vevs are given for $D9$ and $D5$ branes, the results equally apply for D3- and D7-branes since their low-energy effective actions are related by T-duality in the 6 compact dimensions. The resulting expressions for the D3/D7 system, once we relate the auxiliary field vevs to the diverse flux components, lead to the following results

i) ISD $(0,3)$ background

This corresponds to $F_S = 0, F_T \neq 0$. As expected there are no soft terms for fields from 33 open strings. For other fields in 37 and 77 sectors, one obtains

$$
\begin{aligned}
m^2_{\phi^3_{77}} &= m^2_{3/2} \; ; \; m^2_{\phi^1_{77}} = m^2_{\phi^2_{77}} = 0 \qquad (7.9) \\
M^{(77)} &= m_{3/2} \\
A^{(77)}_{ijk} &= A_{\Phi^3_{77}\Phi_{73}\Phi_{37}} = -M^{(77)}
\end{aligned}
$$

[16] Specifically the soft terms correspond to setting $\Theta_i^2 = 1/3$ in that reference and $\sin\theta = 0, 1$ for the cases $(0,3)$ and $(3,0)$ respectively.

Using eq.(7.5) one can check that this is in agreement with our results in sections 3, 4.

ii) IASD $(3,0)$ **background**

This corresponds to $F_T = 0, F_S \neq 0$. As expected there are no soft terms for fields on the $D7$ worldvolume. For the $D3$ and (37) fields one gets

$$\begin{aligned} m^2_{\phi^1_{33}} &= m^2_{\phi^2_{33}} \;=\; m^2_{\phi^3_{33}} \;=\; \frac{V_0}{M_p^2} \\ M^{(33)} &= \frac{V_0^{1/2}}{M_P} \\ A^{(33)}_{ijk} &= A_{\Phi^3_{33}\Phi_{73}\Phi_{37}} \;=\; -M^{(33)} \end{aligned} \tag{7.10}$$

where V_0 is given in (7.6). The results for gaugino and trilinear couplings agree with the results of sections 3, 4 after substituting V_0. However, the results obtained for scalar masses are different. As discussed in [18], this disagreement is not surprising, since our formulae from the local analysis apply to situations with 4d Minkowski space, which in global models are only obtained for ISD backgrounds. Notice also that in the local analysis only Φ^3_{33} is massive, as a consequence of our very restrictive ansatz for the warp factor and 5-form. It is clear that in the presence of additional objects, and indeed in any complete global compactification, the warp factor and 5-form have a non-trivial dependence on all coordinates. This will lead to non-trivial mass terms for all D3-branes scalars, as described in the general ansatz in [18], in agreement with the result obtained using the 4d effective action.

We already mentioned in section 5 that soft terms for ISD backgrounds lead to a positive definite scalar potential (5.6) which includes all bosonic soft terms. This is easy to understand from the effective supergravity scalar potential. The general form of the later is

$$V \;=\; e^K \left(g^{i\bar{j}} (D_i W)(\bar{D}_{\bar{j}} \bar{W}) \;-\; 3|W|^2 \right) \;+\; D - \mathrm{term} \tag{7.11}$$

where the indices run over all the chiral fields and K and W are the Kähler potential and superpotential of the theory. Here the Kähler covariant derivative is given by

$$D_i W \;=\; \partial_i W \;+\; W \partial_i K \tag{7.12}$$

If we have $F_S = 0, F_T \neq 0$ it is easy to see from eq.(7.8) that the only non-vanishing scalar potential left is that due to the auxiliary field of the ϕ^3_{77} chiral multiplet, i.e.

$$V_{ISD} \;=\; e^K \left(g^{3\bar{3}} (D_3 W)(\bar{D}_{\bar{3}} \bar{W}) \right) \tag{7.13}$$

where the index 3 refers to the scalar field ϕ^3_{77}. After rescaling the matter fields to canonical value one gets

$$V_{ISD} \;=\; | \; - M^*_{77}{\Phi^3_{77}}^* \;+\; \partial_{\phi^3} W \; |^2 \tag{7.14}$$

which reproduces the result in (5.6).

We have already noted in different places the apparent symmetry of SUSY breaking soft terms under the exchange D3 $\leftrightarrow$ D7, $G_{\bar{1}\bar{2}\bar{3}} \leftrightarrow G_{123}$. This is easy to understand from the effective action point of view. Consider the gauge kinetic functions and Kähler potential in eq.(7.7) corresponding to D7-branes transverse to the third complex plane and a set of $D3$-branes. Under a T-duality along the first two complex planes one has

$$\begin{array}{ccc} D7_3 & \longleftrightarrow & D3 \\ S & \longleftrightarrow & T_3 \; . \end{array} \tag{7.15}$$

T-duality is thus the origin of the symmetry of the gauge kinetic functions and Kähler potential in (7.7) under those replacements. From this symmetry it is clear that SUSY-breaking by $F_{T_3} \neq 0$ (resp. $F_S \neq 0$) on $D7$-branes leads to the same soft terms as $F_S \neq 0$ (resp. $F_{T_3} \neq 0$) do on $D3$-branes.

7.2 Intersecting D7-branes and ISD fluxes

Up to now we have just considered D7-D3 systems in which one stack of parallel branes is located transverse to the third complex plane. More generally some interesting compactifications may contain different sets of D7-branes which wrap different 4-cycles in a CY, and intersect over complex curves. In these situations there is a new sector of open strings stretched between different D7-brane stacks, and it would be interesting to compute the effect of fluxes on their dynamics. Again we lack a DBI+CS description of these sectors, and moreover the local geometric symmetry (which is reduced to just $SO(2)^3$) is not powerful enough to determine the soft terms. However, in this case the effective supergravity Lagrangian approach described in this section is still valid, and provides a complete well-defined answer.

Let us again consider a simplified situation in which we consider a factorized toroidal compactification on $T^2_1 \times T^2_2 \times T^2_3$. Chiral theories of interest may be later obtained by making some orbifold projection and/or adding magnetic fluxes on the worldvolume of the D7-branes (see [24] for a recent discussion). We will consider three classes of D7-branes, denoted $D7_i$, wrapping the 4-tori transverse to the i^{th} complex plane,

respectively. In addition to the matter fields of in 7_i7_i , 7_i3 and 33 sectors, which we have considered up to now, there are additional massless hypermultiplets Φ_{ij} from the 7_i7_j sectors, for $i \neq j$. Let us consider the case of two different sets of D7-branes $D7_i$, $D7_j$ and a set of D3-branes inside their worldvolume. The 4d superpotential which describes the D3- and D7-branes geometric and Wilson line scalars has the form

$$W_{D7_iD7_j} \;=\; \phi_{ij}\phi_{ji}\phi_{ii}^k \;-\; \phi_{ij}\phi_{ji}\phi_{jj}^k \;+\; \phi_{ij}\phi_{37_i}\phi_{37_j} \tag{7.16}$$

where $k \neq i, j$ and there is no sum on i, j indices. Consider now the addition of a $(0,3)$ ISD background, which corresponds to assuming $F_T \neq 0$, with the rest of the auxiliary fields vanishing. From the general no-scale potential property described above, only the auxiliary fields of matter fields on D7-branes contribute to the scalar potential. Thus we expect for the scalars at the $D7_i - D7_j$ intersections to contribute to the scalar potential as

$$V_{D7_iD7_j} \;\propto\; |-M_{77}^*{\Phi_{ij}}^* + \partial_{\phi_{ij}} W\,|^2 \;=\; |-M_{77}^*{\Phi_{ij}}^* + \phi_{ji}\phi_{ii}^k + \phi_{ji}\phi_{jj}^k + \phi_{37_i}\phi_{37_j}|^2\,. \tag{7.17}$$

Indeed this may be explicitly verified by using the toroidal Kähler potential for $D7_i$-$D7_j$ system described in section 6 of ref.[23] and plugging $F_T \neq 0$, $F_S = 0$. One finds the above result with a coefficient of proportionality =1/2 (corresponding to the power 1/2 in the T-dependence of the metric of the $D7_i$-$D7_j$ fields). Note that the scalars ϕ_{ij} at the $D7_i$-$D7_j$ intersections become massive in the presence of a $(0,3)$ background. Thus, as a general conclusion, one observes that not only the geometrical moduli of each D7-brane are fixed by ISD fluxes, but also the open string moduli associated to the intersections are fixed. This is a natural consequence of these scalars being geometric moduli as well, associated to the possibility of recombining the intersecting branes into a single smooth one, as we briefly discuss in section 8.2

8 Going beyond the $\Sigma_4 = T^4$ case

Although in previous sections we have discussed the computation of the soft terms for D7-branes on $T^4 \times C$, the results are slightly more general. The main simplifying assumptions are given by

i) the metric ansatz, which implicitly assumes that the D7-branes wrap a 4-cycle Σ_4 with a normal tangent bundle, hence the local Calabi-Yau geometry is $\Sigma_4 \times C$. This on the other hand implies that Σ_4 is Ricci flat, and hence it is T^4 or K3.

ii) the fact that the warp factor and 5-form are constant over the 4-cycle. This on the other hand, is not a fundamental restriction, but rather a technical one which

greatly simplifies the computations. As discussed, we expect a varying background to only slightly modify the conclusions obtained for a constant one.

iii) specific profiles for the KK reduction. In particular, we have taken constant profiles for the internal components of the 8d gauge bosons, leading to the Wilson line moduli Φ_1, Φ_2, and constant profile for the zero mode of the 8d scalar Φ_3.

In this section we would like to consider how far one may go in relaxing them, and what results may be obtained at the quantitative or, if not possible, qualitative level.

8.1 D7-branes on K3$\times C$

In this section, we argue that the computation can be extended, with little modification, to the next simplest case, namely D7-branes wrapped on K3 in the local model K3$\times C$ (see remark i)). Indeed, the local geometry is consistent with the ansatz for the background

$$\begin{aligned} ds^2 &= Z_1^{-1/2}(x)\eta_{\mu\nu}dx^\mu dx^\nu + Z_2(x)^{1/2}\, g^{CY}_{mn}\, dx^m dx^m \\ &= Z_1^{-1/2}(x)\,\eta_{\mu\nu}dx^\mu dx^\nu + Z_2(x)^{1/2}\, g^{K3}_{ab}\, dx^a dx^b + Z_2(x)^{1/2}\, dz_3 d\bar{z}_3 \\ \chi_4 &= \chi(x)dx^0dx^1dx^2dx^3 \quad ; \quad F_5 = d\chi_4 + *_{10}d\chi_4 \end{aligned} \tag{8.1}$$

with an expansion (3.11) for the supergravity backgrounds. The background also includes a set of fluxes

$$G_3 \;=\; \beta\wedge dz_3 \,+\, \beta'\wedge d\bar{z}_3 \,+\, \gamma\wedge dz_3 \,+\, \gamma'\wedge d\bar{z}_3 \tag{8.2}$$

On K3, the 2-homology is as follows. There is one $(2,0)$- and one $(0,2)$-form, and 20 (1,1)-forms. The internal product $\omega_1\cdot\omega_2 = \int_{K3}\omega_1\wedge\omega_2$ endows the 22-dimensional vector space with signature $(3,19)$, namely there are 3 selfdual and 19 anti-selfdual 2-forms. The above forms β, β', γ, γ' are linear combinations of these, according to their duality properties, see table 3.1, so the expression encodes a large number of components. However, most of the physics only depends on the duality properties on the 4-cycle and on the CY. Hence although β, β', γ and γ' in the above expression contain a larger number of flux components, the full computation in appendix A and in sections 3, 4 goes through.

Also the equations of motion for the background can be solved consistently by considering the warp factor and 5-form to depend only on the directions z_3, $\bar{z}_3$. The result is also irrelevant for the soft term lagrangian, hence we skip its expression (which is anyway formally given by (3.30), once the latter is translated in terms of G, G', $\mathcal{G}$, $\mathcal{G}'$.

The result for the 8d soft term lagrangian is given by expressions (3.17) and (3.24), with the understanding that G, G', $\mathcal{G}$, $\mathcal{G}'$ correspond to the relevant pieces in (8.2). Similarly, if there are D3-branes present, the soft terms involving the 37 fields are given by (4.10), with the same understanding.

An important point concerns the KK reduction of this 8d action. Since K3 has no harmonic 1-forms, the KK reduction of the internal components of the 8d gauge fields leads to no massless scalars. This can be regarded as a non-trivial global effect arising from the derivative couplings for the 8d gauge fields (included in the dots of the expressions for the 8d action). The mass scale of 4d massive states in the KK tower is $1/R$ (where R is a typical length scale of the 4-cycle), which in the large R regime of validity of the analysis, is much larger than the flux-induced masses whose scale is α/R^3. Hence, all modes arising from the internal components of the gauge bosons are not relevant for the flux physics, and can be ignored in the analysis below. The same conclusion follows for the supersymmetric partners Ψ_1, Ψ_2 of these scalars, which get a large supersymmetric mass due to compactification effects.

On the other hand, there is one massless scalar from the KK reduction of the 8d complex scalar Φ_3. Given the large mass of the massive states in the tower, we may restrict to this zero mode. As easily follows from the independence of the background in the 4-cycle, the internal profile of this zero mode is constant over the K3. Hence, despite the curvature of K3, it is still correct to drop the derivative terms in the 8d action to capture the physics of the massless 4d mode. The final 4d action for 77 fields can therefore be written [17]

$$\begin{aligned}
\mathcal{L} &= \mathrm{Tr}\,(\partial_\mu \Phi^m \partial_\mu \Phi^{\bar{m}} - \qquad (8.3)\\
&- \frac{g_s}{36}\,(\,\mathcal{G}^* \cdot \mathcal{G}^* + (G')^* \cdot (G')^*)\ \Phi^3\Phi^3 + h.c. - \frac{g_s}{18}\,(\,\mathcal{G} \cdot \mathcal{G}^* + G' \cdot (G')^*)\ \Phi^{\bar{3}}\Phi^3 + \\
&- \frac{1}{2} g_{YM}^2 ([\Phi^3, \Phi^{\bar{3}}])^2 + \frac{g_s^{1/2}}{6\sqrt{2}} Tr[\,(\,\lambda\,,\,\Psi^3\,)\, i\sigma_y (G'^* \cdot \vec{\sigma})\begin{pmatrix} \lambda \\ \Psi^3 \end{pmatrix} + h.c.
\end{aligned}$$

This system provides the simplest realization of a D7-brane system where all D7-brane moduli are lifted. The curvature of K3 eliminates the possibility of turning on Wilson lines, while the flux induces a mass term for the moduli associated to transverse motion.

Indeed, this kind of system and the stabilization of all D7-brane moduli has been discussed in [21], from the perspective of F-theory on K3$\times K3$.

[17] As discussed in section 3.5, since in the K3 case the flux backgrounds are not constant, it is understood that the coefficients appearing in the 4d soft term lagrangian correspond to the average value of the flux density over K3, as follows from the KK reduction.

8.2 D7-brane with multiple geometric moduli

As discussed, our analysis does not exactly apply to situations where the 4-cycle has a non-trivial normal bundle. On the other hand, some of the obtained features can be extrapolated at the qualitative level to these other situations. Given that the toroidal case is certainly non-generic concerning the bundle degrees of freedom (e.g. Wilson lines), we momentarily center on aspects related to the scalars parametrizing motion of the D7-brane in the transverse direction.

As discussed in section 3.3.1, there are local geometries such that the 4-cycle is rigid, and there are no 4d scalars associated to D7-brane motion. On the other hand, there are geometries where the 4-cycle has multiple moduli, leading to several 4d complex scalars, which are massless in the absence of fluxes. From the viewpoint of the 8d dynamics, these 4d fields are associated to the zero modes of the laplacian operator for the 8d scalar field $\Phi^3(x^a)$. We center in this latter case, since it is richer from the viewpoint of soft terms.

In the presence of fluxes, one may in principle operate as in section 3, allowing for a more general ansatz for the local metric, mixing directions longitudinal and normal to the 4-cycle. This will induce additional terms in the 8d action, which we have not considered. On the other hand, the terms we have computed are present in these situations as well, so that we again conclude that the 8d scalar Φ^3 acquires an 8d mass term induced from the fluxes. This term radically modifies its KK reduction on Σ_4, in particular eliminating all previously existing zero modes. Hence we conclude that all geometric moduli for D7-branes are stabilized by 3-form fluxes. A more quantitative estimate of these masses can be obtained by splitting the 3-form flux in components G, G', $\mathcal{G}$, $\mathcal{G}'$, according to their duality properties on the threefold and on Σ_4, as done in the case of K3.

We would like to conclude by mentioning that it is relatively simple to consider examples of D7-branes with several geometric moduli. A particularly simple case is to consider a factorized six-torus $T^6 = T^2 \times T^2 \times T^2$, and stacks of D7-branes (labeled by an index a) wrapped on say the third two-torus, times a 2-cycle, product of 1-cycles (n_a^i, m_a^i) in $T^2 \times T^2$ (where $i = 1, 2$). Each D7-brane has one position modulus (plus Wilson line moduli, which we ignore). In addition, two D7-branes intersect a number of times $\prod_i (n_a^i m_b^i - m_a^i n_b^i)$. By suitably choosing the geometry, the configuration is supersymmetric and at each intersection there is a massless hypermultiplet. A particular simple example is to consider two D7-branes along $(1,0) \times (1,0)$ and $(0,1) \times (0,1)$, which, for rectangular two-tori and with some relabeling of coordinates, corresponds

to the configuration of intersecting D7-branes discussed in section 7.2.

Complex scalars in this hypermultiplet parametrize the possibility of recombining the D7-branes into a single smooth one [18]. A convenient local model is provided by two D7-branes, spanning a two-torus times the complex planes $x = 0$ and $y = 0$ respectively. The scalars localized at the intersection point parametrize the possibility of recombining the D7-branes into a D7- on the complex curve

$$xy = \epsilon \tag{8.4}$$

Hence our schematic analysis of D7-branes on 4-cycles with several moduli applies to the interesting case of intersecting D7-branes. The fact that the mentioned moduli get massive in the presence of fluxes is consistent with the results we found in section 7 from the effective lagrangian approach for 7_i7_j fields. Related to this, the fact that the bosonic soft terms for all 77 fields (including 7_i7_j fields) can be encoded in a positive definite scalar potential can be understood from the F/M-theory perspective, since all 77 fields, again including 7_i7_j fields, can be regarded as geometric moduli of the F/M-theory fourfold (the latter fields being associated to recombination of singular loci on the base).

9 Applications

9.1 Local MSSM-like models from D7/D3-branes at orbifolds and soft terms

In this section we present several explicit examples of a local $D7$-$D3$ configurations at orbifolds which lead to a semirealistic models with the SM gauge group (or in fact some extensions thereof) on the worldvolume of $D7$ branes. The geometry is taken to be $(T^4 \times C)/Z_N$ with D7-branes wrapping the orbifolded T^4 and passing through a Z_N singularity in the transverse dimensions. We will show that turning on ISD fluxes leads to an interesting structure of SUSY-breaking soft terms. The interest in having such kinds of local configurations is that, in a second step, we may embed them in a complete F-theory compactification with 4-form fluxes turned on obtaining in this way consistent softly broken $N = 1$ compactifications with semirelistic physics.

[18] This possibility may be constrained from D-term conditions. This represents a global obstruction to the cycle recombination. So not all hypermultiplets at intersections are properly speaking geometric moduli. On the other hand, since the effect of fluxes on fields at intersections is local, we expect it to be insensitive to global obstructions.

Note that in [18] somewhat analogous configurations were constructed [19] , although with SM physics residing on $\overline{D3}$-branes and $\overline{D7}$-branes. This was required in order to obtain non-trivial soft terms for the SM particles in the presence of an ISD background. On the other hand, realizing the SM on D7-branes allows to obtain non-trivial SUSY-breaking soft terms even in ISD backgrounds, as we have discussed in previous sections. The present case is thus more satisfactory, in the sense that the models may be in principle embedded into a complete F-theory compactifications leading to consistent softly broken $N = 1$ theories once fluxes are added.

Let us first consider a local geometry $(T^4 \times C)/Z_3$ with the Z_3 twist given by

$$\theta \quad : \quad (z_1, z_2, z_3) \longrightarrow (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-4\pi i/3} z_3) \tag{9.1}$$

and let us locate a stack of nine $D7$-branes at the orbifold fixed point in the third complex plane. These D7-branes carry twisted CP factors given by

$$\gamma_{\theta,7} \;=\; \text{diag}\,\Big(\mathrm{I}_3, \alpha\;\mathrm{I}_2, \alpha^2\;\mathrm{I}_2, \mathrm{I}_2\Big) \tag{9.2}$$

with $\alpha = \exp(i2\pi/3)$. Let us take a factorized torus $T^4 = T^2 \times T^2$. The Z_3 action leaves $3 \times 3 = 9$ fixed points in T^4, which we will denote by (n, m), with $n, m = 0, \pm 1$. In order to reduce the gauge symmetry let us now add one (quantized) Wilson line with Chan-Paton (CP) matrix

$$\gamma_{W,7} \;=\; \text{diag}\,\Big(\mathrm{I}_7, \alpha, \alpha^2\Big) \tag{9.3}$$

around (say) the second torus. With the addition of this Wilson line the initial gauge group is now broken to $U(3) \times U(2) \times U(2) \times U(1)^2$. Note that now the nine fixed points split into three groups of three each, feeling different D7-brane CP matrices. In particular the points $(n, 0)$, $(n, 1)$ and $(n, -1)$ feel respectively the CP twists $\gamma_{\theta,7}$, $\gamma_{\theta,7}\gamma_{W,7}$ and $\gamma_{\theta,7}\gamma_{W,7}^{-1}$.

Our configuration should be consistent with local tadpole cancellation conditions, which in the present Z_3 twist are given by

$$Tr\gamma_{\theta,7} \;+\; 3Tr\gamma_{\theta,3} \;=\; 0 \tag{9.4}$$

where $\gamma_{\theta,3}$ refers to the CP matrices of possible D3-branes which may reside at the nine orbifold singularities. The above conditions guarantee the cancellation of gauge anomalies in the effective field theory. One can easily check that the matrices $\gamma_{\theta,7}\gamma_{W,7}$

[19] The basic ingredients and techniques for model building with branes at singularities are discussed in [35] (see also [36]), based on the basic results in e.g. [37].

and $\gamma_{\theta,7}\gamma_{W,7}^{-1}$ are traceless, so that tadpole cancellation conditions is satisfied at the six fixed points $(n,1)$ and $(n,-1)$ without the need of adding any D3-brane on them. On the other hand $\mathrm{Tr}\,\gamma_{\theta,7} = 3$ and one needs to add D3-branes at the three fixed points $(n,0)$ in order to cancel tadpoles. The simplest choice is to add two D3-branes at each of the three fixed points, with CP matrices

$$\gamma_{\theta,3_n} \;=\; \mathrm{diag}\,\left(\alpha,\alpha^2\right) \quad ,n=0,\pm 1 \tag{9.5}$$

which verify $\mathrm{Tr}\,\gamma_{\theta,3_n} = -1$. This completes the description of the local brane configuration.

Although the apparent total gauge group is $U(3)\times U(2)\times U(2)\times U(1)^2$ from $D7$-branes and $U(1)^3\times U(1)^3$ from $D3$-branes, it turns out that most of the $U(1)$'s become massive due to the standard generalized Green-Schwarz mechanism by which pseudoanomalous $U(1)$'s combine with RR twisted fields and become massive [38]. An anomaly-free combination corresponding to $U(1)_{B-L}$ remains massless.Thus the actual gauge group is the simplest left-right symmetric extension of the gauge group of the SM [20], with gauge group $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ [21]

From the 77 sector one gets chiral multiplets transforming with respect to $SU(3)\times SU(2)\times SU(2)\times U(1)_{B-L}$ like

$$Q_L^i \;=\; (3,2,1)_{1/3}\;;\; Q_R^i \;=\; (\bar{3},1,2)_{-1/3}\;;\; H^i \;=\; (1,2,2)_0 \quad ; \quad i=1,2,3\;. \tag{9.6}$$

We thus get three generations of quarks together with three sets of Higgs multiplets. From each of the three 3_n7 sectors one gets chiral multiplets

$$\begin{aligned} L^n &= (1,2,1)_{-1}\;;\; R^n \;=\; (1,1,2)_1 \\ D^n &= (3,1,1)_{-2/3}\;;\; \bar{D}^n \;=\; (\bar{3},1,1)_{2/3} \end{aligned} \tag{9.7}$$

plus four additional gauge singlets. Finally from each 33 sector one gets three more singlets. In total the spectrum is that of a Standard Model with three generations of quarks, leptons and Higgsses. There are in addition three sets of vector-like coloured particles $D^n,\bar{D}^n$, which may in fact become massive if one of the singlets from the 33 sector gets a vev. It would be interesting to study the viability of this model, but here we will be more interested in the structure of SUSY-breaking soft terms which may be induced by the presence of fluxes.

[20] One can similarly construct 3-generation models with the SM gauge group. We have chosen here a left-right symmetric example because the set of CP matrices is simpler.

[21] Plus possibly some additional anomaly-free $U(1)$'s, which we ignore in what follows.

Note that the spectrum and perturbative interactions of this type of model is simply a Z_3 projection of the $N = 4$ and $N = 2$ actions considered in previous section for the case of D7-branes wrapping 4-tori. Thus the relevant superpotential terms are as follows:

$$\begin{aligned} W_{77} &= \epsilon_{ijk} Q_L^i Q_R^j H^k \\ W_{37} &= Q_L^3 (\sum_n L^n \bar{D}^n) \ + \ Q_R^3 (\sum_n R^n D^n) \end{aligned} \tag{9.8}$$

Now let us assume we add a $(0,3)$ ISD background $G_{\bar{1}\bar{2}\bar{3}}$. From the results we obtained in section 5 the following non-vanishing soft terms are obtained for gaugino masses, scalar masses and trilinear terms

$$\begin{aligned} M_3 = M_L = M_R = M_{B-L} = M &= \frac{g_s^{1/2}}{3\sqrt{2}} \, (G_{\bar{1}\bar{2}\bar{3}})^* \\ m^2_{Q_L^3} \ = \ m^2_{Q_R^3} \ = \ m^2_{H^3} &= |M|^2 \\ A_{77} \ = \ A_{37} &= -h \ M \end{aligned} \tag{9.9}$$

where here h denotes the superpotential couplings as in eq. (9.8). Note that all soft terms are determined by a single parameter M and that only one generation of quarks gets soft scalar masses. This is not however a serious problem, since, once gaugino masses are present, the rest of the scalars will get a mass from one-loop diagrams with gauginos in the loop.

In this particular 3-generation model the masses of squarks are not universal, which in fully realistic models may lead to phenomenological problems with too much flavor changing neutral currents (FCNC). Furthermore the Z_3 projection does not allow for explicit supersymmetric masses (a μ-term) which is one of the ingredients of the MSSM. These are properties of this particular model, but not generic properties of the flux-induced soft terms.

To exemplify this fact let us now construct a different local D7/D3-brane configuration in which both features, universal squark/slepton masses and a μ-term, appear. The example is of the Pati-Salam type $SU(4) \times SU(2)_L \times SU(2)_R$ which is sufficient for our purposes.

We will be very sketchy in the presentation since we just want to emphasize only the above mentioned facts. We start now with a local geometry $(T^4 \times C)/Z_4$ with the Z_4 twist given by

$$\theta \quad : \quad (z_1, z_2, z_3) \longrightarrow (e^{2\pi i/4} z_1, e^{2\pi i/4} z_2, e^{\pi i} z_3) \tag{9.10}$$

and locate a stack of twelve $D7$-branes at the orbifold fixed point in the third complex plane. These $D7$-branes carry twisted CP factors given by

$$\gamma_{\theta,7} \ = \ \mathrm{diag}\ \Big(\mathrm{I}_4, \alpha^3\ \mathrm{I}_2, \alpha\ \mathrm{I}_2, \mathrm{I}_2, -\mathrm{I}_2\Big) \tag{9.11}$$

with $\alpha = \exp(i2\pi/4)$. Let us take again a factorized torus $T^4 = T^2 \times T^2$. The Z_4 action leaves $2 \times 2 = 4$ fixed points in T^4, which we will denote by (n, m), with $n, m = 0, 1$. We add now one Wilson line with CP matrix

$$\gamma_{W,7} \ = \ \mathrm{diag}\ (\mathrm{I}_8, -\mathrm{I}_2, \mathrm{I}_2) \tag{9.12}$$

around (say) the second torus. The local tadpole cancellation conditions have now the general form $Tr\gamma_{\theta,7} + 2Tr\gamma_{\theta,3} = 0$ for each θ-twisted sector. It is easy to check that all local tadpoles are canceled if we locate four D3-branes at each of the two fixed points $(n, 0)$ with $\gamma_{\theta,3_n} \ = \ \mathrm{diag}\ (\alpha, \alpha^3, -\mathrm{I}_2)$ for $n = 0, 1$. The D7 gauge group is $U(4) \times U(2)_L \times U(2)_R \times U(2)^2$ and there are chiral multiplets from the 77 sector as follows

$$\begin{aligned} F_L^i \ &= \ (4, \bar{2}, 1) \ \ ; \ \ F_R \ = \ (\bar{4}, 1, 2) \ \ ; \ \ i = 1, 2 \\ H \ &= \ (1, \bar{2}, 2) \ \ ; \ \ \bar{H} \ = \ (1, 2, \bar{2}) \end{aligned} \tag{9.13}$$

Thus there are two standard quark/lepton generations corresponding to the first two complex planes. From the third (transverse) complex plane, we get Higgs doublets able to trigger electroweak symmetry breaking, and with Yukawa couplings $\epsilon_{ij}\bar{H}F_L^iF_R^j$ to quarks and leptons. We will not display the spectrum from (37) sectors which just give vector-like multiplets with respect to the Pati-Salam symmetry. We now turn on ISD backgrounds corresponding to $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$. From the results we obtained in section 3 and 5 the following non-vanishing soft terms are obtained

$$\begin{aligned} M_4 \ &= \ M_L = M_R = M \ = \ \frac{g_s^{1/2}}{3\sqrt{2}}\ (G_{\bar{1}\bar{2}\bar{3}})^* \\ m^2_{H,\bar{H}} \ &= \ |M|^2 \ \ ; \ \ \mu \ = \ -\frac{g_s^{1/2}}{6\sqrt{2}}(S_{\bar{3}\bar{3}})^* \\ A_{77} \ &= \ -h\ M \ \ ; \ \ B \ = \ M\ \mu \end{aligned} \tag{9.14}$$

where μ is a SUSY mass for $H, \bar{H}$. Although this Z_4 model has only two generations, the above set of soft terms is quite simple and predictive and have a number of interesting properties. All soft terms are determined by the fluxes $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$ or, alternative by the two parameters M, μ. The squark/slepton masses are universal and equal to zero. This poses no phenomenological problem since they all get large masses at the

one-loop level, as has been abundantly analyzed in the SUSY literature. On the other hand, due to its universality FCNC transitions are suppressed. As we said, one of the interesting aspects is that both a μ- and a B-term, which are important ingredients in the MSSM are obtained, with the simple prediction $B = M\mu$.

The above examples show that ISD fluxes on D7/D3-brane systems not only are promising in order to stabilize the dilaton and complex structure moduli, but also in order to obtain consistent semirealistic Type IIB backgrounds in which fluxes induce SUSY-breaking soft terms of possible phenomenological interest. A comment is in order concerning the scale of the SUSY-breaking soft terms. In analogy with the case of D3-branes, they have a typical scale of α'/R^3, with R a typical length of the compactification manifold. In situations with homogeneously distributed fluxes, this may be recast as M_s^2/M_P. As discussed in [18] the soft term scale may be as low as the weak scale by taking the string scale around the intermediate scale $M_s = 10^{11}$ GeV. Another possibility would be to exploit the effects of a inhomogeneous warping in the internal space to obtain alternative suppression factors.

9.2 Application to KKLT de Sitter vacua

As mentioned in the introduction, another application of our results is related to the proposal in [11], specifically to the proposed mechanism to achieve stabilization of Kähler moduli. This should occur due to non-perturbative contributions to the superpotential (depending on Kähler moduli), arising from strong infrared dynamics on D7-brane gauge sectors or from euclidean D3-brane instantons. A potential problem in generating non-perturbative superpotentials from the formed is that 7-branes may contain too much massless charged matter. Our results for soft terms involving D7-brane matter fields show that the presence of ISD fluxes generally give masses to all the D7-brane geometric moduli. This indicates that generically D7-brane vector-like matter is massive, thus allowing such non-perturbative superpotentials to appear. One explicit example is provided by our K3 example in section 8.1, where in the absence of fluxes the D7-brane gauge theory is pure $N = 2$ SYM, while after introducing supersymmetric fluxes one is left with pure $N = 1$ SYM, which develops a gaugino condensate. This is also in agreement with recent results derived from the F-theory perspective in [21], and from generalized calibrations [22].

We would like to mention that, as discussed in [21], an analogous mechanism may occur concerning the second source of non-perturbative superpotentials, namely euclidean D3-brane instantons. An euclidean D3-brane instanton (a D3-brane wrapped

on a 4-cycle) may contribute to the superpotential if if has two fermion zero modes. In the standard case, this is achieved if the wrapped 4-cycle, once lifted to M/F-theory, leads to a 6-cycle of arithmetic genus equal to one [39]. However, this argument ignores possible effects of the flux on the euclidean D3-brane, which could change the situation as follows. Consider a D3-brane instanton wrapped on a 4-cycle leading, in the absence of fluxes, to a larger number of fermion zero modes. In the presence of fluxes, the instanton action may contain flux-induced terms, lifting pairs of such fermions zero modes, and leaving just two in suitable situations. Such configurations would then contribute to the superpotential.

We would like to point out that, beyond the superficial analogy with the flux effects on D7-branes (which also wrap 4-cycles, etc), there is a deep connection between the two non-perturbative contributions to the superpotential, and the role of flux effects in both situations. Namely, instantons on the D7-brane gauge theory can be regarded as euclidean D3-branes wrapped on the D7-brane 4-cycle (and dissolved into the D7-brane). Hence certain non-perturbative effects on the D7-branes can be described in terms of such euclidean D3-branes. For instance, the gaugino condensate of a D7-brane $N = 1$ $SU(n)$ gauge theory can be understood in terms of fractional instantons, described by euclidean D3-branes whose 6-cycles in the F/M-theory picture are given by exceptional divisors of the elliptic fibration degeneration (i.e. the base 4-cycle times a collapsed 2-sphere in the elliptic fibration degeneration). Thus, the computation of the lifting of D7-brane massless matter, leaving an infrared gauge theory with strong dynamics, contains the implicit computation of the lifting of euclidean D3-brane instanton fermion zero modes from flux-induced effects.

It would be very nice to carry out the computation of the flux-induced effects on euclidean D3-branes more explicitly. We hope the techniques reported in this paper are useful in this direction.

We would like to end by making a few comments about the extent to which our results for soft terms apply to the scenario in [11] to construct deSitter vacua in string theory. In the original proposal there are essentially three ingredients:

- One first compactifies Type IIB theory on a CY orientifold and adds $(0, 3)$ ISD fluxes. The superpotential induced by the flux fixes the dilaton and all complex structure moduli. The theory has a no-scale structure with a vanishing cosmological constant.

- In a second step a modulus dependent superpotential $W(T)$ is introduced. This breaks the no-scale structure and gives rise to an isolated supersymmetric anti

de-Sitter minimum, with negative cosmological constant. The modulus T is thus fixed.

- Finally the cosmological constant is increased to a (possibly tiny) de Sitter value, by adding a set of anti-D3-branes in the bulk (for other possible sources of positive tension, see e.g. [40]).

The kind of soft terms we have computed apply to the theory obtained after the first step only. In principle, using the effective supergravity Lagrangian approach and including a non-perturbative superpotential $W(T)$ one can also compute soft terms in the KKLT vacua. The results however will depend on the form of $W(T)$ as well as on the Kähler potential and gauge kinetic function of the effective theory. Furthermore, there will be additional dependence on the specific way by which we increase the vacuum energy up to de Sitter space. Preliminary work seems to show that, depending on how all these details work out, our specific results for soft terms will apply or not to de-Sitter vacua with the ingredients in [11]. We postpone a systematic discussion of these issues for future work.

10 Final comments

Flux compactifications have been introduced as a canonical mechanism to lead to stabilization a large number of moduli in string compactifications. Interestingly, besides addressing this long-standing problem in string theory, they provide an extremely tractable mechanism to break supersymmetry in string theory, in a computable regime. In fact, despite the seemingly complicated structure of the models, it is possible to carry out detailed computations of the flux-induced soft SUSY-breaking terms in gauge sectors localized on the volume of D-branes in the models.

In this paper we have described the effects of field strength 3-form fluxes on configurations of D3/D7-branes, with D7-branes wrapped on 4-cycles, in Calabi-Yau compactifications. The computation and results turn out to be richer and more interesting than the similar effects on D3-branes, discussed in [18, 19].

We have carried out a complete analysis of flux-induced soft terms on D7-branes in local Calabi-Yau geometries, by expanding the DBI+CS world-volume action coupled to the flux supergravity background. For the more complicated sector of open strings stretched between D3 (or anti-D3) branes and D7-branes, where a similar general action

is not known, we have succeeded in exploiting the symmetries of the system to fully determine the soft terms.

We have emphasized that in D3/D7-brane systems, a prominent role in determining the 4d physics is played by the 4-cycle on which the D7-branes wrap. Although our computations have been explicitly carried out only in the case of D7-branes wrapped on T^4, the correct identification of the underlying physical interpretation of the soft terms has allowed us to extrapolate the results to other situations, like D7-branes on K3, or on 4-cycles with multiple geometric moduli.

The results markedly differ from the situation with just D3-branes. Indeed, we have found that gauge sectors on D7-branes acquire non-trivial soft terms even in the presence of ISD 3-form fluxes. This is interesting, since such fluxes lead to consistent string compactifications to 4d Minkowski space without any runaway potential for Kähler moduli. Hence the models constitute 4d classical string vacua with zero cosmological constant and non-trivial soft terms for the gauge sector on D7-branes.

This implies no contradiction whatsoever with the no-scale structure of the 4d effective action. Indeed, we have exactly reproduced the soft terms computed in the local analysis from the 4d effective action, where the flux background components are identified with auxiliary fields of the different moduli chiral multiplets.

This is an improvement over the situation with just D3-branes, where, in order to obtain non-trivial soft terms, one must either consider D3-branes in IASD fluxes (which in compact models lead to runaway potentials for Kähler moduli), or anti-D3-branes in ISD fluxes (where now the backreaction of the antibranes would induce the potential).

The set of four-dimensional gauge sectors which can be constructed using D3/D7-brane configurations is extremely rich, and includes semirealistic models, which are nevertheless very tractable (being orbifold projections of toroidal and flat space models). Therefore, we believe that D3/D7-brane configurations embedded in 3-form flux backgrounds provide a well-defined and very interesting starting point for phenomenological studies of SUSY-breaking in string theory. Further discussions in this direction have been initiated in [20].

Finally we have also emphasized that our results have an important application to the recent proposal in [11] to stabilize all moduli. In particular, flux effects lifting charged vector-like matter on D7-brane world-volume help in generating non-perturbative superpotentials for Kähler moduli.

Clearly, many questions remain open. For instance, it would be interesting to evaluate the impact on soft terms of additional ingredients possibly present in more

involved compactifications (like non-perturbative superpotentials for Kähler moduli, or additional sources of tension). It is very tempting to speculate that, since D-branes are only sensitive to the local background around them, and our analysis has considered a fairly general such local background (with 4d Poincare invariance being the only strong assumption), our results will be valid for any model with small enough cosmological constant (much smaller than the flux scale).

Also it would be interesting to extend our analysis to more involved configurations of D7-branes, for instance D7-branes carrying non-trivial gauge bundles on their world-volume. In the toroidal setup, such models are dual to configurations of intersecting D6-branes, which have been extensively studied for phenomenological model building. Hence, a detailed analysis of the resulting soft terms for these configurations would very much improve the possible phenomenological applications of fluxes.

We expect much progress in these and other directions, in the interesting interplay of D-branes and fluxes, in particular concerning supersymmetry breaking and soft terms.

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A Computation of the D=8 action and dimensional reduction in a T^4.

In this section we present the detailed computation of the expressions (3.17) and (3.19). The conventions that we will adopt are exactly the same than in [18], summarized in the appendix therein.

As we mentioned in the main text, our starting point is Myers' action for a D7-brane written in the Einstein's frame

$$\begin{aligned} S = \quad & -\mu_7 \int d^8\xi \, STr\left[e^{-\phi} \sqrt{-\det(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] \, + \, \sigma F_{\mu\nu}) \, \det(Q_{ij})} \right] \\ & + \mu_7 g_s \int STr\left(P\left[\sigma C_6 F_2 + C_8 - C_6 B_2\right]\right) \end{aligned}$$

where in the CS action we have kept only pieces giving contributions to the soft terms. For our particular background $P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}]$ and $Q^i_{\ j}$ are given by

$$\begin{aligned} P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] &= P[g_{\mu\nu}g_s^{1/2} - B_{ab}\delta^a_\mu\delta^b_\nu + \delta^a_\mu B_{am}(Q^{-1}-\delta)^{mn}B_{nb}\delta^b_\nu] \\ Q^i_{\ j} &= \delta^i_{\ j} + i\sigma[\Phi^i,\Phi^k](G_{kj}g_s^{1/2} - B_{kj}) \end{aligned}$$

We start by computing the first determinant in the DBI piece of the action. Neglecting derivative couplings, it can be factorized between the Minkowski and the 4-cycle pieces as

$$\begin{aligned} & det(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] + F_{\mu\nu}) = \\ = \; & det(g_{\mu\nu}g_s^{1/2} + 2g_s^{1/2}\sigma^2 D_\mu\Phi^3 D_\nu\Phi^{\bar{3}} + \sigma F_{\mu\nu}) \cdot det(g_s^{1/2} + \sigma F_{ab} - B_{ab} + \\ & +2g_s^{1/2}\sigma^2 D_a\Phi^3 D_b\Phi^{\bar{3}} + B_{am}(Q^{-1}-\delta)^{mn}B_{nb} - \sigma B_{3(a}D_{b)}\Phi^3 - \sigma B_{\bar{3}(a}D_{b)}\Phi^{\bar{3}}) \end{aligned} \tag{A.1}$$

with a, b running over the internal coordinates of Σ_4 and

$$D_a\Phi^m = \partial_a\Phi^m + i[A_a,\Phi^m] = i[A_a,\Phi^m] \tag{A.2}$$

so there can still be possible contributions from the covariant derivative in the non-abelian case.

Ignoring derivative couplings is justified because the relevant fields in the two cases we center on, namely $T^4 \times C$ and K3$\times C$, have constant profiles in the 4-cycle, as explained in the main text. It is however straightforward (but lengthy) to take them into account to obtain the complete 8d action, hence we do not provide the result.

We can expand now these two determinants making use of the formula

$$det(1+M) = 1 + Tr\ M - \frac{1}{2}Tr\ M^2 + \frac{1}{2}(Tr\ M)^2 + \dots \tag{A.3}$$

where the dots in this and in further expressions refer to contributions giving rise to derivative couplings or couplings with dimension higher than four in the low energy effective action.

Therefore,

$$\begin{aligned} & det(P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^{ij} E_{j\nu}] + \sigma F_{\mu\nu}) = \\ = \; & -g_s^4 [Z_1^{-1}(\Phi^3, \Phi^{\bar{3}}) Z_2(\Phi^3, \Phi^{\bar{3}})]^2 + 2 g_s^4 \sigma^2 \partial_\mu \Phi^3 \partial_\mu \Phi^{\bar{3}} - g_s^3 \frac{1}{2}(B_{ab} - \sigma F_{ab})(B_{\bar{a}\bar{b}} - \sigma F_{\bar{a}\bar{b}}) - \\ - \; & g_s^4 \sigma^2 F_{\mu a} F^{\mu a} + \sigma^2 g_s^{9/2} [A_a, \Phi^3][A^a, \Phi^{\bar{3}}] + i g_s^4 \sigma (B_{3a}[A^a, \Phi^3] + B_{\bar{3}a}[A^a, \Phi^{\bar{3}}]) + \ldots \end{aligned} \tag{A.4}$$

The last term of this expression will survive to the KK reduction only in cases on which the Wilson line moduli and Φ^3 have profiles with different parity. This is not the case in compactifications on 4-cycles trivially fibered in the normal direction. Therefore, in what follows we will include as well these terms in the final dots.

The second determinant of the DBI piece is much simpler to compute

$$det(Q^i_j) = 1 + \sigma^2([\Phi^3, \Phi^{\bar{3}}])^2 g_s \tag{A.5}$$

Putting everything together and Taylor expanding the square root, we have the following eight dimensional action

$$\begin{aligned} \mathcal{L} \; = \; & \mu_7 g_s STr\, Z_1^{-1}(\Phi^3, \Phi^{\bar{3}}) Z_2(\Phi^3, \Phi^{\bar{3}}) \Big(1 + \sigma^2 \partial_\mu \Phi^3 \partial_\mu \Phi^{\bar{3}} - \\ & - \frac{1}{2}(B_2|_{\Sigma_4} - \sigma F_2) \wedge *_4 (B_2|_{\Sigma_4} - \sigma F_2) + \frac{\sigma^2}{2} F_{\mu a} F^{\mu a} - \\ & - \frac{1}{2} g_s \sigma^2 ([\Phi^3, \Phi^{\bar{3}}])^2 - \frac{1}{2} g_s \sigma^2 [A^a, \Phi^3][\Phi^{\bar{3}}, A^{\bar{a}}] + (\sigma C_6 \wedge F_2 + C_8 - C_6 \wedge B_2)|_{\Sigma_4} + \ldots \Big) \end{aligned} \tag{A.6}$$

It only remains to plug our background in this expression. From the equations of motion of Type IIB supergravity (see e.g. [41]) one realizes that C_6 and C_8 (in our conventions) are given by

$$\begin{aligned} dC_6 \; &= \; H_3 \wedge \left(C_4 + \frac{1}{2} B_2 \wedge C_2 \right) - *Re G_3 \\ dC_8 \; &= \; H_3 \wedge C_6 - *Re\; d\tau \end{aligned} \tag{A.7}$$

Decomposing G_3 as in (3.1) and making use of these expressions, we can integrate the relevant RR and NS field strengths for our particular background [22] to obtain

$$C_6|_{M_4 \times \Sigma_4} \; = \; -\frac{1}{6i}((\beta - \beta^*) z^3 - (\beta' - \beta'^*) \bar{z}^3 - (\gamma - \gamma^*) z^3 + (\gamma' - \gamma'^*) \bar{z}^3) \wedge d\, Vol_{4d} + \ldots$$

[22] We have chosen a gauge on which all the coordinates transverse to Minkowski are treated in the same way.

$$
\begin{aligned}
C_8|_{M_4\times\Sigma_4} &= -\frac{g_s}{36}\Big((\beta z^3+\gamma'\bar z^3-\beta'^*\bar z^3-\gamma^* z^3)\wedge(\beta z^3+\gamma'\bar z^3-\beta'^*\bar z^3-\gamma^* z^3)- \\
&\quad (\beta'\bar z^3+\gamma z^3-\beta^* z^3-\gamma'^*\bar z^3)\wedge(\beta'\bar z^3+\gamma z^3-\beta^* z^3-\gamma'^*\bar z^3)\Big)\wedge d\,Vol_{4d} \\
B_2|_{\Sigma_4} &= -\frac{g_s}{6i}\Big((\beta-\beta^*)z^3+(\beta'-\beta'^*)z^{\bar 3}+(\gamma-\gamma^*)z^3+(\gamma'-\gamma'^*)z^{\bar 3}\Big) \qquad \text{(A.8)}
\end{aligned}
$$

with $(\beta^*)_{mn}=(\beta'_{\bar m\bar n})^*$, etc.

Plugging these expressions in (A.7) we finally get

$$
\begin{aligned}
\mathcal{L} &= \mu_7 g_s\sigma^2 STr\, Z_1^{-1}(\Phi^3,\Phi^{\bar 3})Z_2(\Phi^3,\Phi^{\bar 3})\Big(\frac{1}{\sigma^2}+\partial_\mu\Phi^3\partial_\mu\Phi^{\bar 3}+ \qquad \text{(A.9)} \\
&\quad -\frac{g_s}{36}(2\gamma^*\wedge *_4\gamma'\Phi^{\bar 3}\Phi^3+2\beta^*\wedge *_4\beta'\Phi^{\bar 3}\Phi^3-\gamma'\wedge *_4\gamma'\Phi^{\bar 3}\Phi^{\bar 3}-\beta'\wedge *_4\beta'\Phi^{\bar 3}\Phi^{\bar 3}+h.c.)|_{\Sigma_4} \\
&\quad -\frac{g_s}{3}(\beta'_{ab}\Phi^{\bar 3}A^aA^b+\gamma'_{ab}\Phi^{\bar 3}A^aA^b+h.c.)- \\
&\quad -\frac{g_s^{-1}}{4}F_{ab}F_{\bar a\bar b}+\frac{1}{2}g_s^{-1}F_{\mu a}F_{\mu\bar a}-\frac{1}{2}g_s([\Phi^3,\Phi^{\bar 3}])^2-\frac{1}{2}g_s[A^a,\Phi^3][\Phi^{\bar 3},A^{\bar a}]+\dots\Big)
\end{aligned}
$$

which in terms of the SU(3) irreducible representations of Table 1 can be rewritten as

$$
\begin{aligned}
\mathcal{L} &= \mu_7 g_s\sigma^2 STr Z_1^{-1}(\Phi^3,\Phi^{\bar 3})Z_2(\Phi^3,\Phi^{\bar 3})(\frac{1}{\sigma^2}+\partial_\mu\Phi^3\partial_\mu\Phi^{\bar 3}- \qquad \text{(A.10)} \\
&- \frac{g_s}{18}(\frac{1}{4}(S_{12})^2+\frac{1}{4}(A_{12})^2-\frac{1}{2}G_{\bar 1\bar 2\bar 3}S_{\bar 3\bar 3}-\frac{1}{4}S_{22}S_{11})\Phi^{\bar 3}\Phi^{\bar 3}+h.c.- \\
&- \frac{g_s}{18}(|G_{\bar 1\bar 2\bar 3}|^2+\frac{1}{4}|S_{\bar 3\bar 3}|^2+\frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2+|A_{ij}|^2))\Phi^{\bar 3}\Phi^3+ \\
&+ \sum_{j,k,p=1,2}\frac{g_s}{3}\epsilon_{3jk}((S_{kp})^*+(A_{kp})^*)\Phi^3A^{[j}A^{\bar p]}+h.c.- \\
&- \frac{g_s}{6}\epsilon_{ij3}S_{\bar 3\bar 3}A^iA^j\Phi^{\bar 3}+h.c.-\frac{2g_s}{3}G_{\bar 1\bar 2\bar 3}\Phi^{\bar 3}A^{[\bar 1}A^{\bar 2]}+h.c.- \\
&- g_s[A^a,A^{(b}][A^{\bar b)},A^{\bar a}]+\partial_\mu A^a\partial_\mu A^{\bar a}-\frac{1}{2}g_s([\Phi^3,\Phi^{\bar 3}])^2-\frac{1}{2}g_s[A^a,\Phi^3][A^{\bar a},\Phi^{\bar 3}])\ +\ \dots
\end{aligned}
$$

We can perform now the dimensional reduction in the simplest case of $\Sigma_4=T^4$. Since on this case the profiles of the fields are constant over the 4-torus, the derivative couplings will not give any contribution in the four dimensional action, consistently with our above considerations. Therefore, the integration over T^4 becomes trivial

$$
\begin{aligned}
\mathcal{L}_{4d} &= \frac{\mathcal{V}}{(2\pi)^5(\alpha')^2}STr\Big(-\frac{g_s}{18}\Big(\frac{1}{4}(S^*_{\bar 1\bar 2})^2+\frac{1}{4}(A^*_{\bar 1\bar 2})^2-\frac{1}{2}G^*_{123}S^*_{33}-\frac{1}{4}S^*_{\bar 2\bar 2}S^*_{\bar 1\bar 1}\Big)\Phi^3\Phi^3+h.c.- \\
&\quad -\frac{g_s}{18}\Big(|G_{\bar 1\bar 2\bar 3}|^2+\frac{1}{4}|S_{\bar 3\bar 3}|^2+\frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2+|A_{ij}|^2)\Big)\Phi^{\bar 3}\Phi^3+ \qquad \text{(A.11)} \\
&\quad +\sum_{j,k,p=1,2}\frac{g_s}{3}\epsilon_{3jk}((S_{kp})^*+(A_{kp})^*)\Phi^3\Phi^{[j}\Phi^{\bar p]}+h.c.- \\
&\quad -\frac{g_s}{6}\epsilon_{ij3}S_{\bar 3\bar 3}\Phi^i\Phi^j\Phi^{\bar 3}+h.c.-\frac{2g_s}{3}G_{\bar 1\bar 2\bar 3}\Phi^{\bar 3}\Phi^{[\bar 1}\Phi^{\bar 2]}+h.c.- \\
&\quad -g_s[\Phi^a,\Phi^{(b}][\Phi^{\bar b)},\Phi^{\bar a}]+\partial_\mu\Phi^a\partial_\mu\Phi^{\bar a}\Big)
\end{aligned}
$$

where $\mathcal{V}$ is the volume of T^4.

Finally, rescaling all the fields by $(2\pi)^{5/2}\alpha'\mathcal{V}^{-1/2}$ in order to have canonical kinetic terms, we get the four dimensional action

$$\begin{aligned}
\mathcal{L}_{4d} \;=\; & Tr\,(\partial_\mu\Phi^m\partial_\mu\Phi^{\bar m}- \qquad\qquad (A.12)\\
& -\frac{g_s}{18}\left(\frac{1}{4}[(S_{12})^*]^2+\frac{1}{4}[(A_{12})^*]^2-\frac{1}{2}(G_{\bar 1\bar 2\bar 3})^*(S_{\bar 3\bar 3})^*-\frac{1}{4}(S_{22})^*(S_{11})^*\right)\Phi^3\Phi^3+h.c.-\\
& -\frac{g_s}{18}\left(|G_{\bar 1\bar 2\bar 3}|^2+\frac{1}{4}|S_{\bar 3\bar 3}|^2+\frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2+|A_{ij}|^2)\right)\Phi^{\bar 3}\Phi^3-\\
& -\sum_{k,p=1,2}\frac{g_s^{1/2}g_{YM}}{6}\epsilon_{ijk}((S_{kp})^*+(A_{kp})^*)\Phi^i\Phi^j\Phi^{\bar p}+h.c.+\\
& +\frac{g_s^{1/2}g_{YM}}{6}\epsilon_{ij3}S_{\bar 3\bar 3}\Phi^i\Phi^j\Phi^{\bar 3}+h.c.+\frac{g_s^{1/2}g_{YM}}{9}G_{\bar 1\bar 2\bar 3}\epsilon_{\bar i\bar j\bar k}\Phi^{\bar i}\Phi^{\bar j}\Phi^{\bar k}+h.c.-\\
& -g_{YM}^2[\Phi^i,\Phi^{(j}][\Phi^{\bar j)},\Phi^{\bar i}]\Big)
\end{aligned}$$

with g_{YM} given by

$$g_{YM}^2 = g_s(2\pi)^5(\alpha')^2\mathcal{V}^{-1} \qquad (A.13)$$

References

[1] A. Strominger, 'Superstrings with torsion', Nucl. Phys. B274 (1986) 253.
J. Polchinski, A. Strominger, 'New vacua for type II string theory', Phys. Lett. B388 (1996) 736, hep-th/9510227;
J. Michelson, 'Compactifications of type IIB strings to four-dimensions with non-trivial classical potential', Nucl. Phys. B495 (1997) 127, hep-th/9610151.

[2] K. Becker, M. Becker, 'M theory on eight manifolds', Nucl. Phys. B477 (1996) 155, hep-th/9605053.

[3] S. Gukov, C. Vafa, E. Witten, 'CFT's from Calabi-Yau four folds', Nucl. Phys. B584 (2000) 69, Erratum-ibid. B608 (2001) 477, hep-th/9906070

[4] K. Dasgupta, G. Rajesh, S. Sethi, 'M theory, orientifolds and G-flux', JHEP 9908 (1999) 023, hep-th/9908088.

[5] T. R. Taylor, C. Vafa, 'R R flux on Calabi-Yau and partial supersymmetry breaking', Phys. Lett. B474 (2000) 130, hep-th/9912152;
S. Gukov, 'Solitons, superpotentials and calibrations', Nucl. Phys. B574 (2000) 169, hep-th/9911011;

K. Behrndt, S. Gukov, ‘Domain walls and superpotentials from M theory on Calabi-Yau three folds’, Nucl. Phys. B580 (2000) 225, hep-th/0001082.

[6] S. B. Giddings, S. Kachru, J. Polchinski, ‘Hierarchies from fluxes in string compactifications’, hep-th/0105097.

[7] B. R. Greene, K. Schalm, G. Shiu, ‘Warped compactifications in M and F theory’, Nucl. Phys. B584 (2000) 480, hep-th/0004103;
G. Curio, A. Klemm, D. Lust, S. Theisen, ‘On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H fluxes’, Nucl. Phys. B609 (2001) 3, hep-th/0012213;
M. Haack, J. Louis, ‘M theory compactified on Calabi-Yau fourfolds with background flux’, Phys. Lett. B507 (2001) 296, hep-th/0103068;
J. Louis, A. Micu, ‘Type 2 theories compactified on Calabi-Yau threefolds in the presence of background fluxes’, hep-th/0202168.

[8] S. Kachru, M. Schulz, S. Trivedi, ‘Moduli stabilization from fluxes in a simple iib orientifold’, hep-th/0201028.
A. R. Frey, J. Polchinski, ‘N=3 warped compactifications’, hep-th/0201029.
P. K. Tripathy, S. P. Trivedi, ‘Compactification with flux on K3 and tori’, hep-th/0301139.

[9] R. D’Auria, Sergio Ferrara, S. Vaula, ‘N=4 gauged supergravity and a IIB orientifold with fluxes’, New J.Phys. 4 (2002) 71, hep-th/0206241;
S. Ferrara, M. Porrati, ‘N=1 no-scale supergravity from IIB orientifolds’, Phys. Lett. B545 (2002) 411, hep-th/0207135;
R. D’Auria, S. Ferrara, M. Lledó and S. Vaulá, ’No-scale $N = 4$ supergravity coupled to Yang-Mills: the scalar potential and super-Higgs effect’, Phys.Lett.B557 (2003) 278, hep-th/0211027;
R. D’Auria, S. Ferrara, F. Gargiulo, M. Trigiante and S. Vaulá, ’$N = 4$ supergravity Lagrangian for Type IIB on T^6/Z_2 orientifold in presence of fluxes and D3-branes’, JHEP 0306:045,2003 , hep-th/0303049;
M. Berg, M. Haack, B. Kors, ‘An Orientifold with fluxes and branes via T duality’, Nucl. Phys. B669 (2003) 3, hep-th/0305183.
L. Andrianopoli, S. Ferrara and M. Trigiante, ’Fluxes, supersymmetry breaking and gauged supergravity’, hep-th/0307139.

[10] S. Sethi, C. Vafa, E. Witten, 'Constraints on low dimensional string compactifications', Nucl. Phys. B480 (1996) 213, hep-th/9606122.

[11] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, 'De Sitter Vacua in String Theory', Phys.Rev.D68(2003)046005, hep-th/0301240.

[12] F. Denef, M. R. Douglas, B. Florea, 'Building a better racetrack', JHEP 0406 (2004) 034, hep-th/0404257.

[13] D. Robbins, S. Sethi, 'A Barren landscape', hep-th/0405011.

[14] R. Blumenhagen, D. Lust, T. R. Taylor, 'Moduli Stabilization in Chiral Type IIB Orientifold Models with Fluxes', hep-th/0303016.

[15] J. F. G. Cascales, A. M. Uranga, 'Chiral 4d N = 1 string vacua with D branes and NSNS and RR fluxes', JHEP 0305 (2003) 011, hep-th/0303024; 'Chiral 4-D string vacua with D-branes and moduli stabilization', hep-th/0311250.

[16] F. Marchesano, G. Shiu, L.-T. Wang, to appear.

[17] M. Graña, 'MSSM parameters from supergravity backgrounds', hep-th/0209200.

[18] P.G. Cámara, L.E. Ibáñez and A. Uranga, 'Flux-induced SUSY-breaking soft terms', hep-th/0311241, Nucl.Phys.B689 (2004) 195.

[19] M. Grana, T. W. Grimm, H. Jockers and J. Louis, "Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes," Nucl. Phys. B **690**, 21 (2004) [arXiv:hep-th/0312232].

[20] L.E. Ibáñez, "The fluxed MSSM", hep-ph/0408064.

[21] L. Goerlich, S. Kachru, P. Tripathy, S. Trivedi, 'Gaugino condensation and nonperturbative superpotentials in F-theory', hep-th/0407130.

[22] J.F.G. Cascales, A.M. Uranga, 'Branes on generalized calibrated submanifolds', hep-th/0407132.

[23] L. E. Ibáñez, C. Muñoz and S. Rigolin, 'Aspects of Type I string phenomenology', Nucl. Phys. B553 (1999) 43, hep-ph/9812397.

[24] D. Lust, S. Reffert and S. Stieberger, "Flux-induced soft supersymmetry breaking in chiral type IIb orientifolds with D3/D7-branes," arXiv:hep-th/0406092.

[25] J. Louis, H. Jockers et al, to appear.

[26] M. Graña, J. Polchinski, 'Supersymmetric three form flux perturbations on AdS_5', Phys. Rev. D63 (2001) 026001, hep-th/0009211.

[27] D. S. Freed, E. Witten, 'Anomalies in string theory with D-branes', hep-th/9907189.

[28] E. Witten, 'Baryons and branes in anti-de Sitter space', JHEP 9807 (1998) 006, hep-th/9805112.

[29] J.M. Maldacena, G. W. Moore, N. Seiberg, 'Geometrical interpretation of D-branes in gauged WZW models', JHEP 0107 (2001) 046, hep-th/0105038.

[30] M. Cederwall, A. von Gussich, B. E. Nilsson, P. Sundell and A. Westerberg, 'The Dirichlet super-p-branes in ten-dimensional type IIA and IIB supergravity', Nucl. Phys. B **490** (1997) 179, hep-th/9611159;
M. Cederwall, A. von Gussich, B. E. Nilsson and A. Westerberg, 'The Dirichlet super-three-brane in ten-dimensional type IIB supergravity', Nucl. Phys. B **490** (1997) 163, hep-th/9610148;
M. Graña, 'D3-brane action in a supergravity background: The fermionic story', Phys. Rev. D **66** (2002) 045014, hep-th/0202118;
D. Marolf, L. Martucci and P. J. Silva, 'Actions and fermionic symmetries for D-branes in bosonic backgrounds', JHEP **0307** (2003) 019, hep-th/0306066.

[31] A. Lawrence, J. McGreevy, 'Local string models of soft supersymmetry breaking', JHEP 0406 (2004) 007, hep-th/0401034.

[32] L.E. Ibáñez and D. Lüst, 'Duality anomaly cancellation, minimal string unification and the effective low-energy Lagrangian of 4-D strings', Nucl. Phys. B382 (1992) 305, hep-th/9202046.

[33] V. S. Kaplunovsky, J. Louis, 'Model independent analysis of soft terms in effective supergravity and in string theory, Phys. Lett. B306 (1993) 269, hep-th/9303040.

[34] A. Brignole, L. E. Ibáñez, C. Muñoz, 'Towards a theory of soft terms for the supersymmetric Standard Model', Nucl. Phys. B422 (1994) 125, Erratum-ibid. B436 (1995) 747, hep-ph/9308271.

[35] G. Aldazabal, L. E. Ibáñez, F. Quevedo, A.M. Uranga, 'D-branes at singularities: A Bottom up approach to the string embedding of the standard model', JHEP 0008 (2000) 002, hep-th/0005067.

[36] D. Berenstein, V. Jejjala and R.G. Leigh, 'The standard model on a D-brane', Phys.Rev.Lett.88 (2002) 071602, hep-ph/0105042;
L. F. Alday and G. Aldazabal, "In quest of just the Standard Model on D-branes at a singularity," hep-th/0203129.

[37] M. R. Douglas, G. W. Moore, 'D-branes, quivers, and ALE instantons', hep-th/9603167.

[38] A. Sagnotti, 'A Note on the Green-Schwarz mechanism in open string theories', Phys. Lett. B294 (1992) 196, hep-th/9210127. L. E. Ibanez, R. Rabadan and A. M. Uranga, "Anomalous U(1)'s in type I and type IIB D = 4, N = 1 string vacua," Nucl. Phys. B **542** (1999) 112 [arXiv:hep-th/9808139].

[39] E. Witten, 'Nonperturbative superpotentials in string theory', Nucl. Phys. B474 (1996) 343, hep-th/9604030.

[40] C. P. Burgess, R. Kallosh, F. Quevedo, 'De Sitter string vacua from supersymmetric D terms', JHEP 0310 (2003) 056, hep-th/0309187.
A. Saltman and E. Silverstein, 'The scaling of the no scale potential and De Sitter model building', hep-th/0402135.

[41] J. Polchinski, M. J. Strassler, 'The String dual of a confining four-dimensional gauge theory', hep-th/0003136.